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# Agent-Based Simulation in Complex Networks

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Miguel Rebollo (@mrebollo)

# Session 5. Dynamics

*Complex networks display  
a surprising degree of robustness:  
although key components malfunction,  
local failures rarely lead  
to the loss of the global information.*

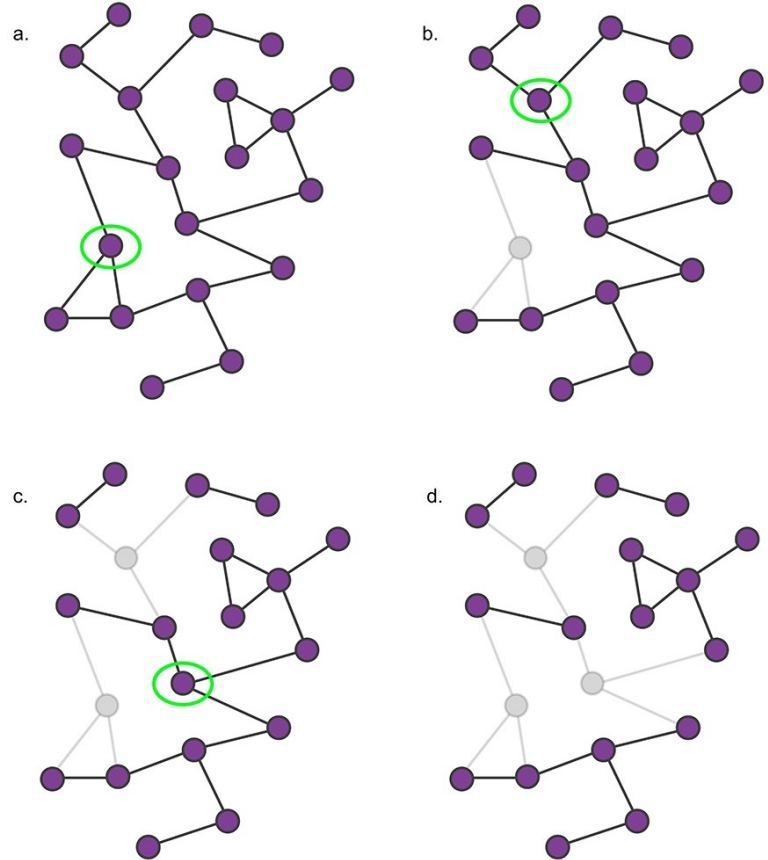
*We'll study why this effect appears.*

# PERCOLATION THEORY

*Impact of node removal*

# Percolation theory

removal of one link has limited impact, but remove several can break a network into components. How many nodes we have to delete?

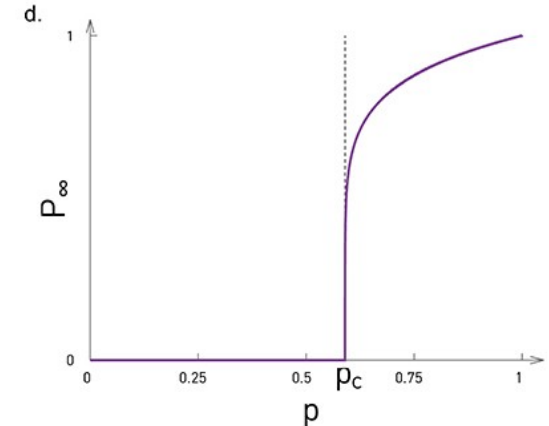
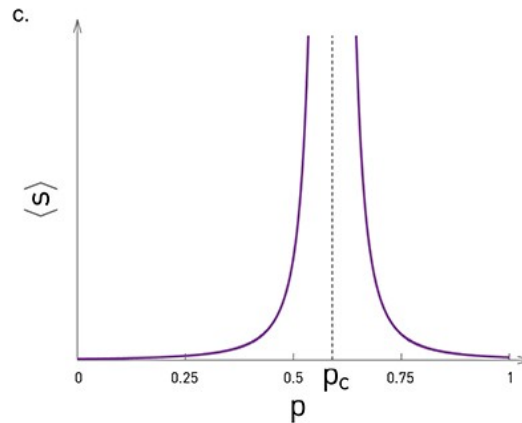
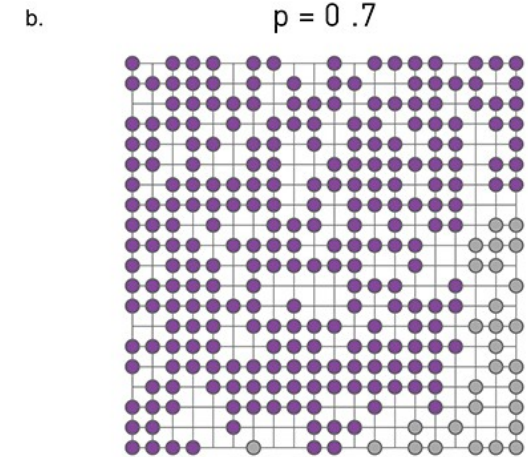
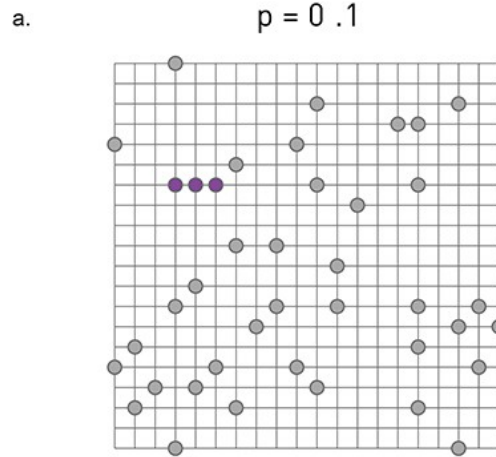


# Percolation

Which is the expected size of the largest cluster?

What is the average cluster size?

When it is formed?



# Percolation

Average cluster size  $\langle s \rangle$

$$\langle s \rangle \sim |p - p_c|^{-\gamma}$$

Order parameter  $P_\infty$

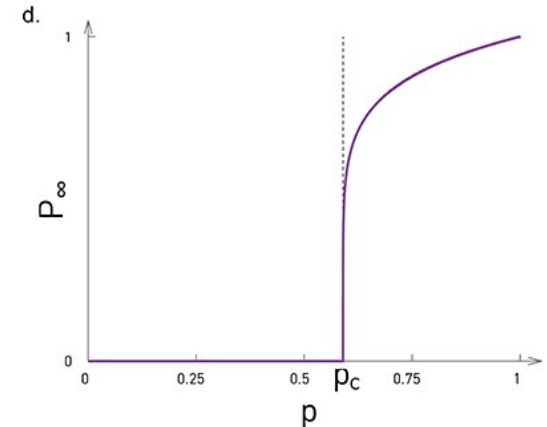
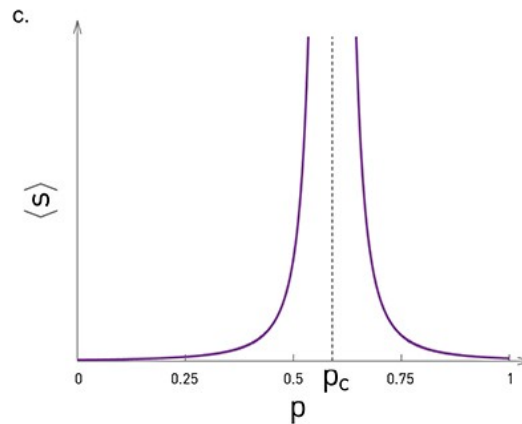
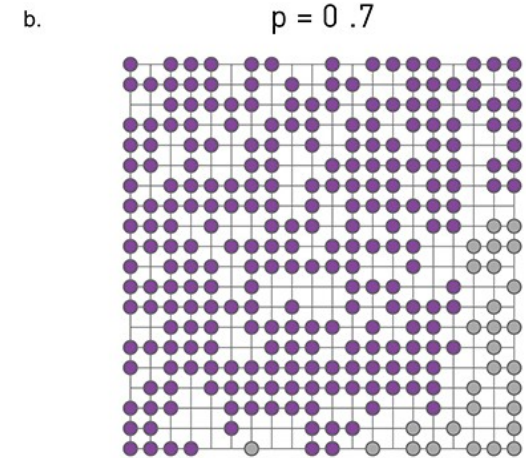
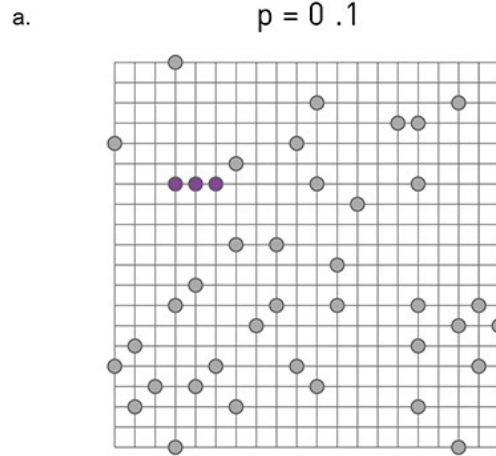
$$P_\infty \sim (p - p_c)^\beta$$

Correlation length

$$\xi \sim |p - p_c|^{-\nu}$$

$\gamma$ ,  $\beta$  and  $\nu$ :  
critical exponents

$p_c = 0.593$  universal value



# ROBUSTNESS

*Robust (from oak -roble-) system that resist errors  
and failures without degrading*

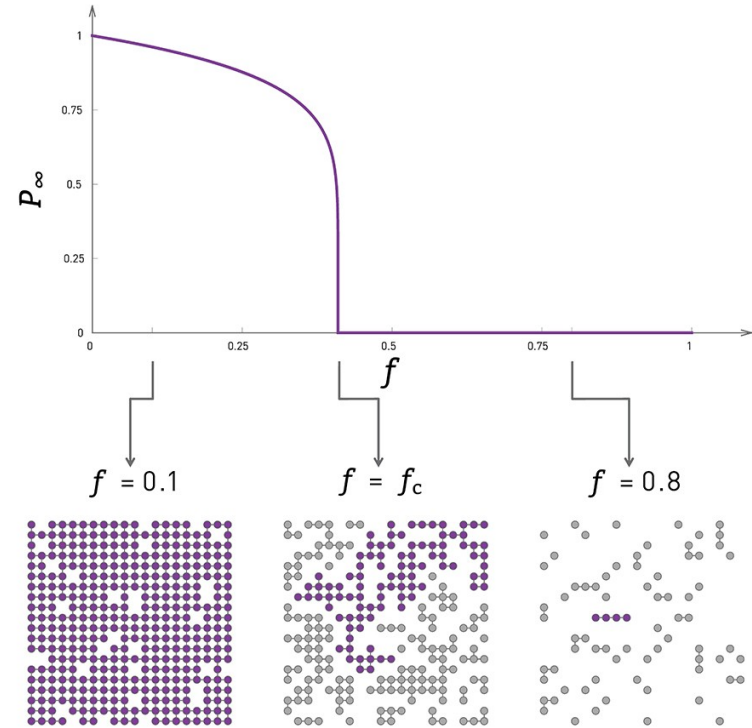


# Robustness

Inverse percolation: node removal until disconnection

Critical exponents are the same

Fragmentation process is abrupt, not gradual



$$0 < f < f_c :$$

There is a giant component.

$$P_\infty \sim |f - f_c|^\beta$$

$$f = f_c :$$

The giant component vanishes.

$$f > f_c :$$

The lattice breaks into many tiny components.

# Efficiency

$$E = \frac{1}{N(N-1)} \sum_{i,j \neq i} \frac{1}{d_{ij}}$$

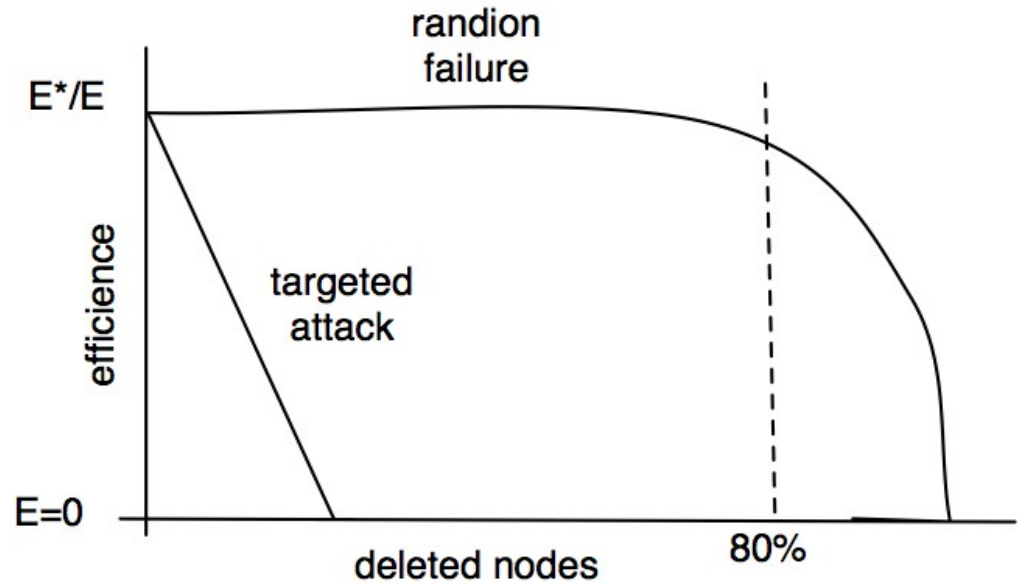
The bigger the distance, the less efficient the network

Networks with short paths are more efficient

# Vulnerability

Measures how failures affect to the efficiency (variation of eff when a link/node disappears)

Dependence on the topology



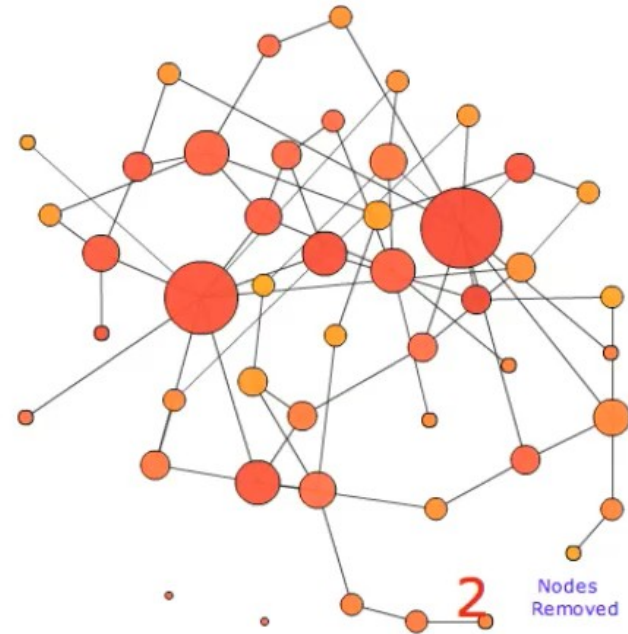
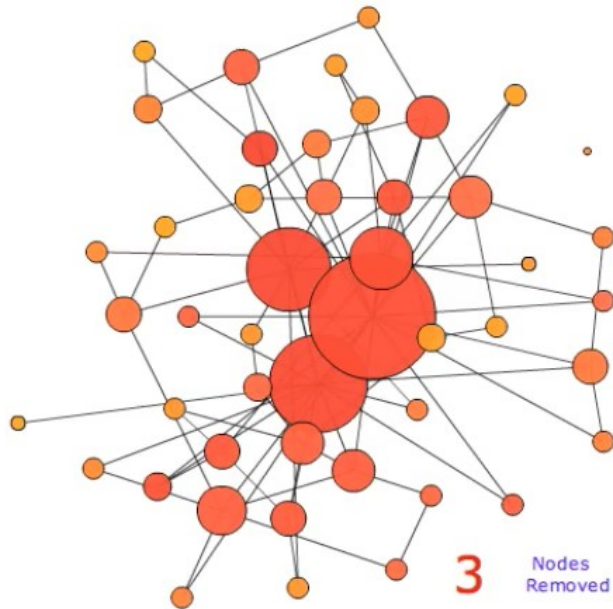
# ATTACK TOLERANCE

*What happens in the network when s sabotage is intended?*

# Deliberate Attacks

Random

Scale-free

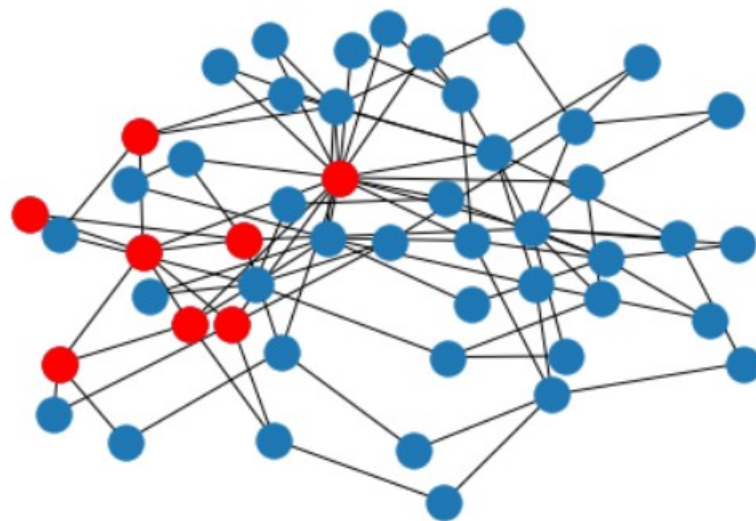
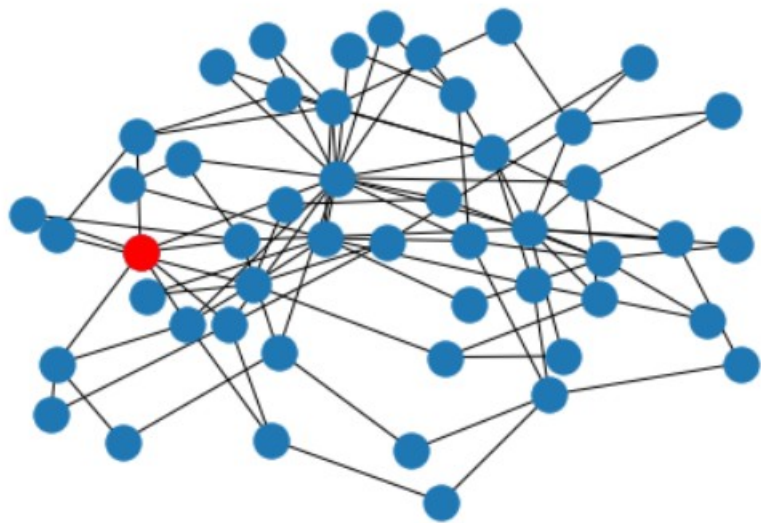


Hubs play a significant role in failure tolerance

# CASCADE FAILURES

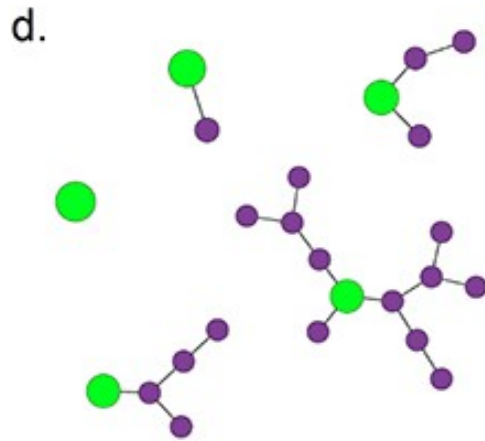
*Because failures do not arrive isolated*

# Cascades



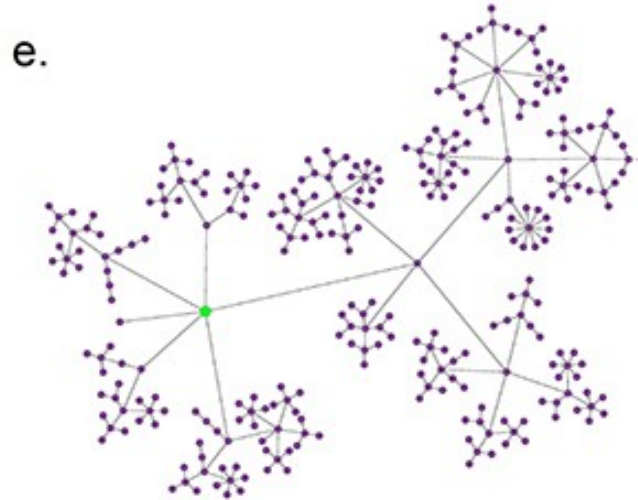
# Three regimen

SUBCRITICAL



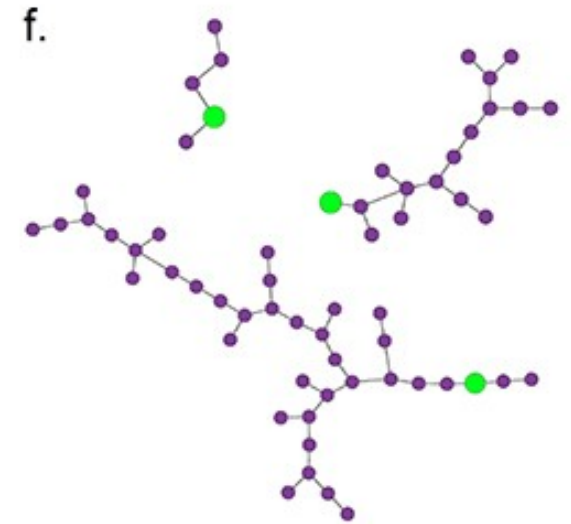
Subcritical  $\langle k \rangle < 1$

SUPERCRITICAL



Supercritical  $\langle k \rangle > 1$

CRITICAL



Critical  $\langle k \rangle = 1$



# DIFFUSION

*Diseases, computer viruses, innovation or memes,  
all they share the same transmission scheme*

# Closing credits. Planet of the Apes



# EPIDEMIC MODELING

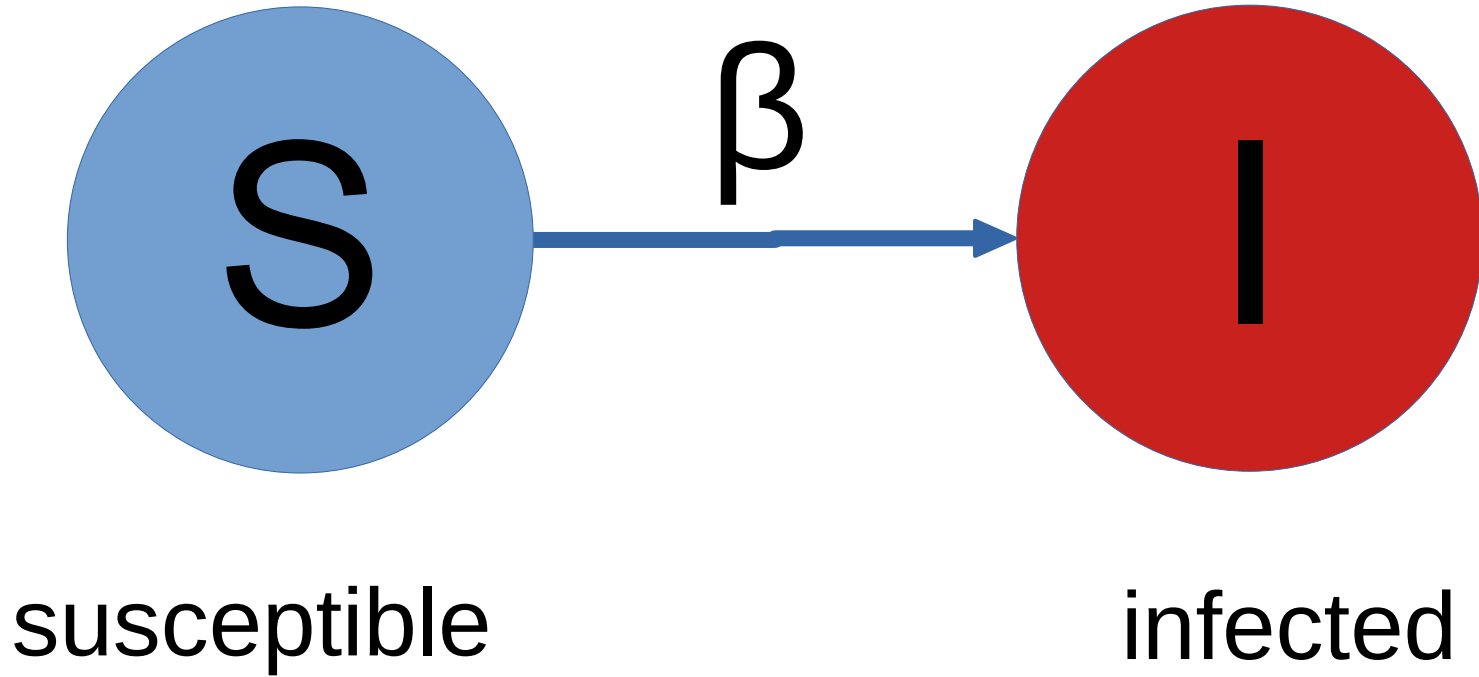
*Mathematical models that simulate how a disease evolves*

# Based on two concepts

**compartmentalization:** classification of individuals in groups according to their state (SI / SIS / SIR)

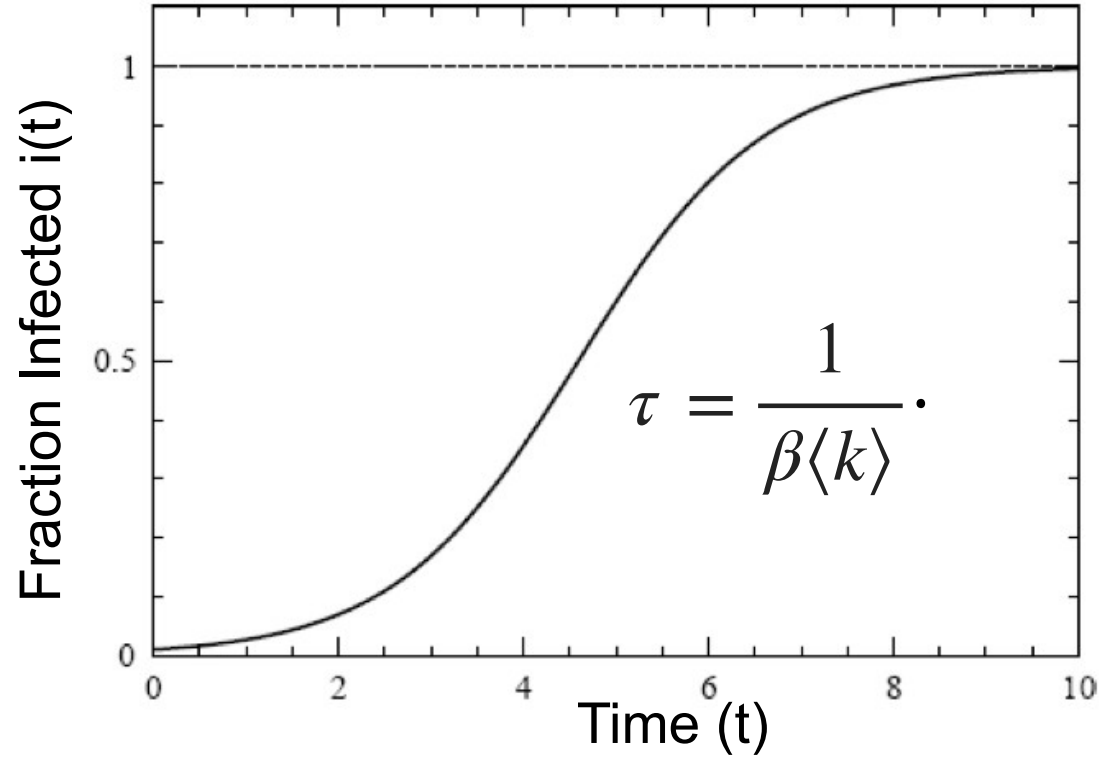
**homogenous mixing:** all individuals have same prob. to get infected (no contact network)

# SI model

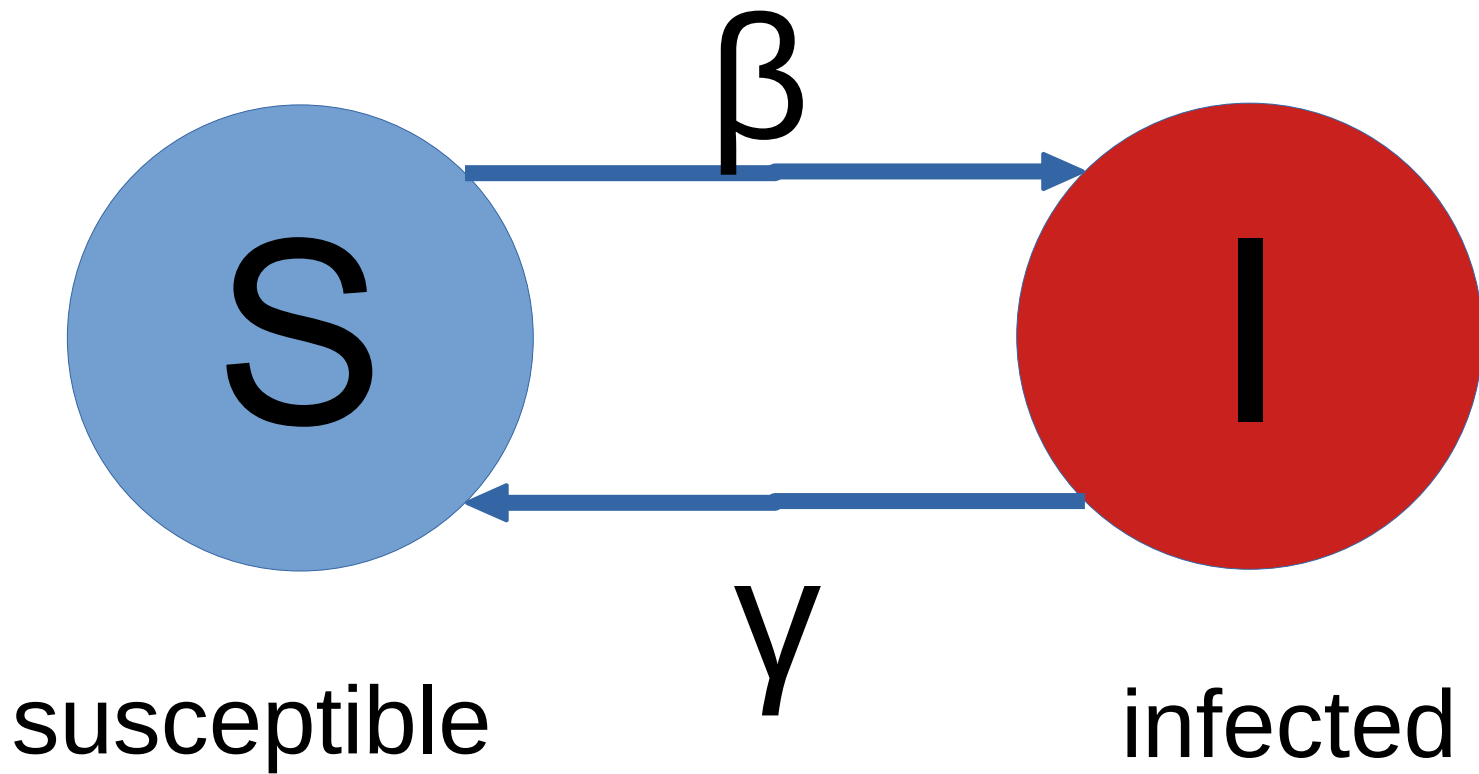


# SI model

Infection  
increases until  
all population  
infected



# SIS model



# SIS model

Two regimenes

Endemic state ( $\gamma < \beta$ )

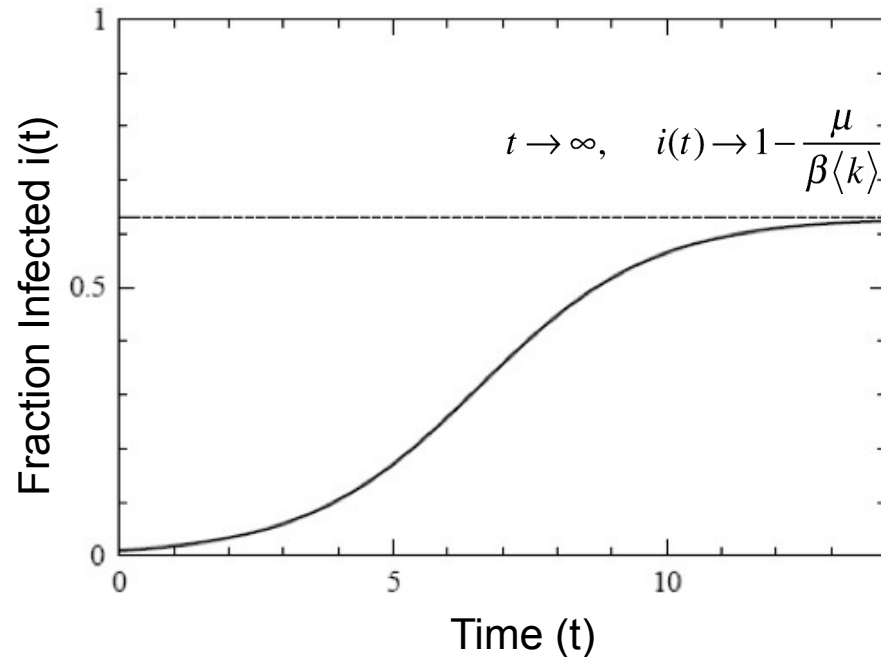
Disease free ( $\gamma > \beta$ )

$$\tau = \frac{1}{\gamma(R_0 - 1)}$$

$$R_0 = \frac{\beta \langle k \rangle}{\gamma}$$

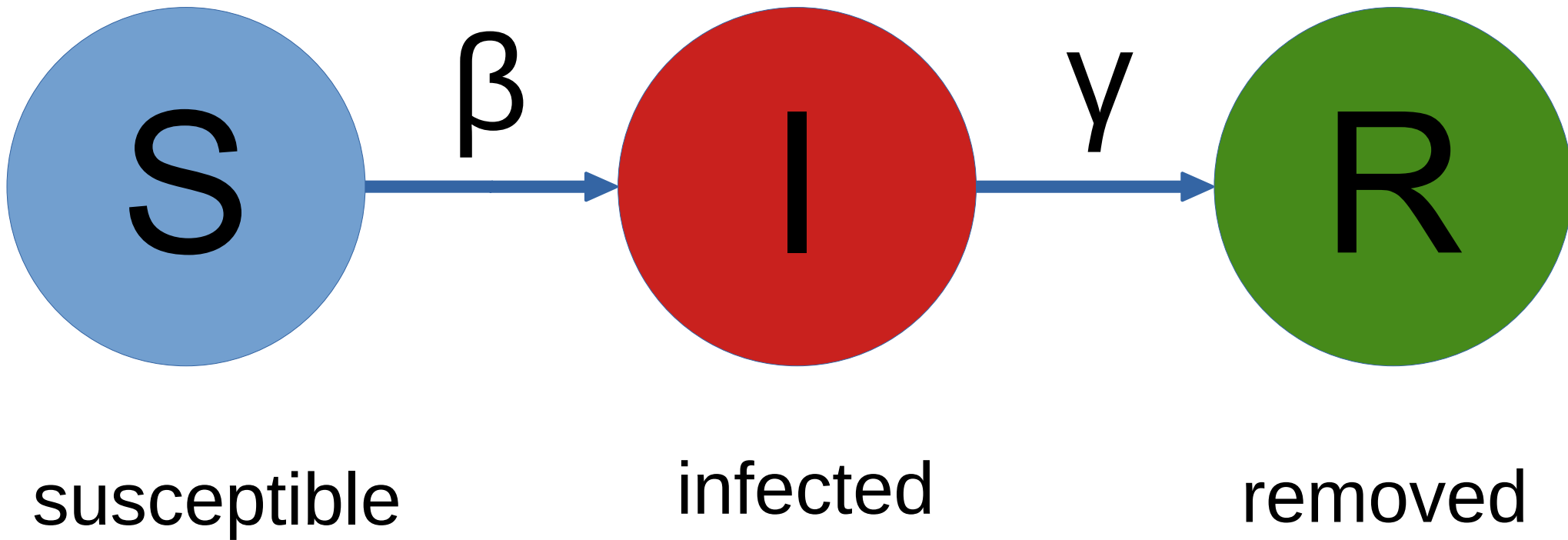
Characteristic time

Reproductive number  
< 1 extinction  
> 1 propagation



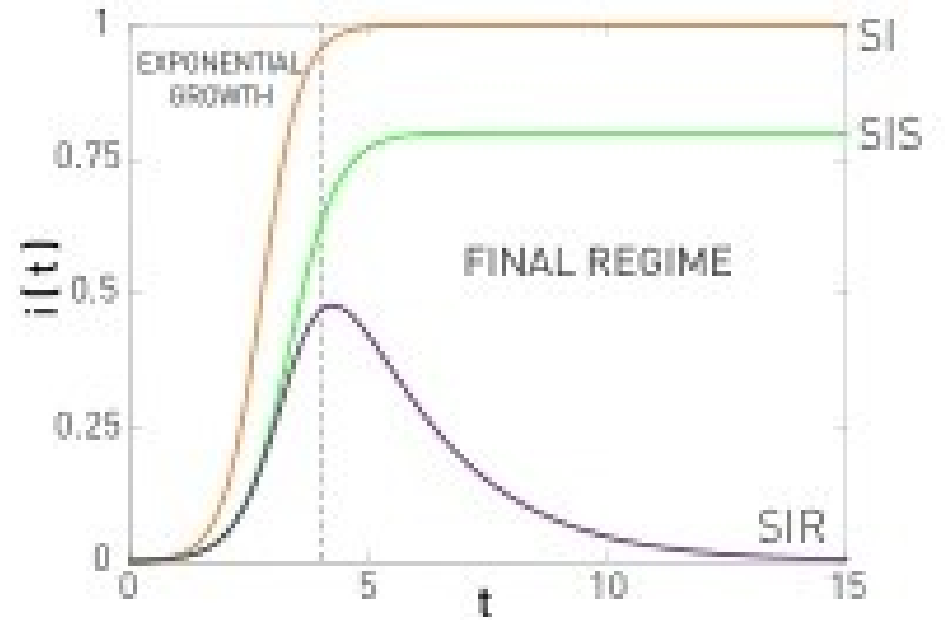
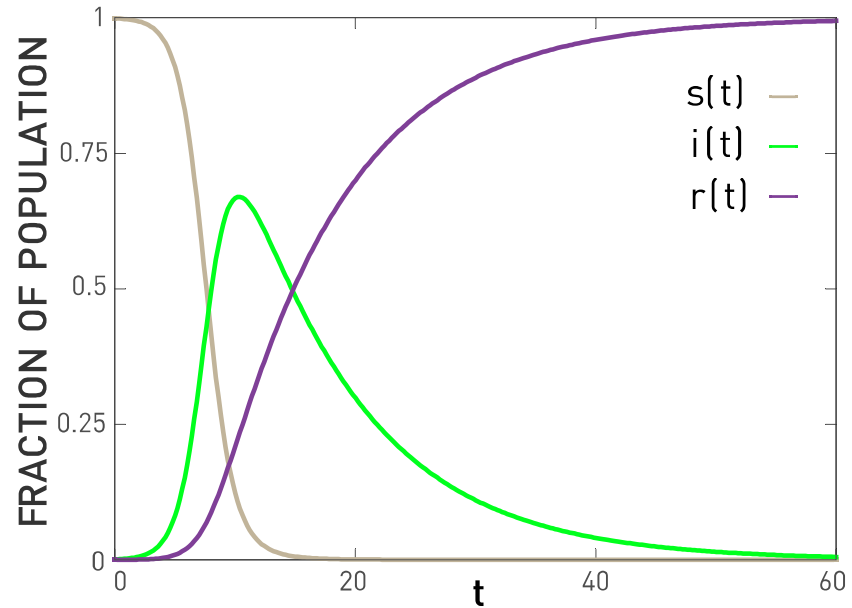


# SIR model



# SIR model

(c)



# EPIDEMIC IN NETWORKS

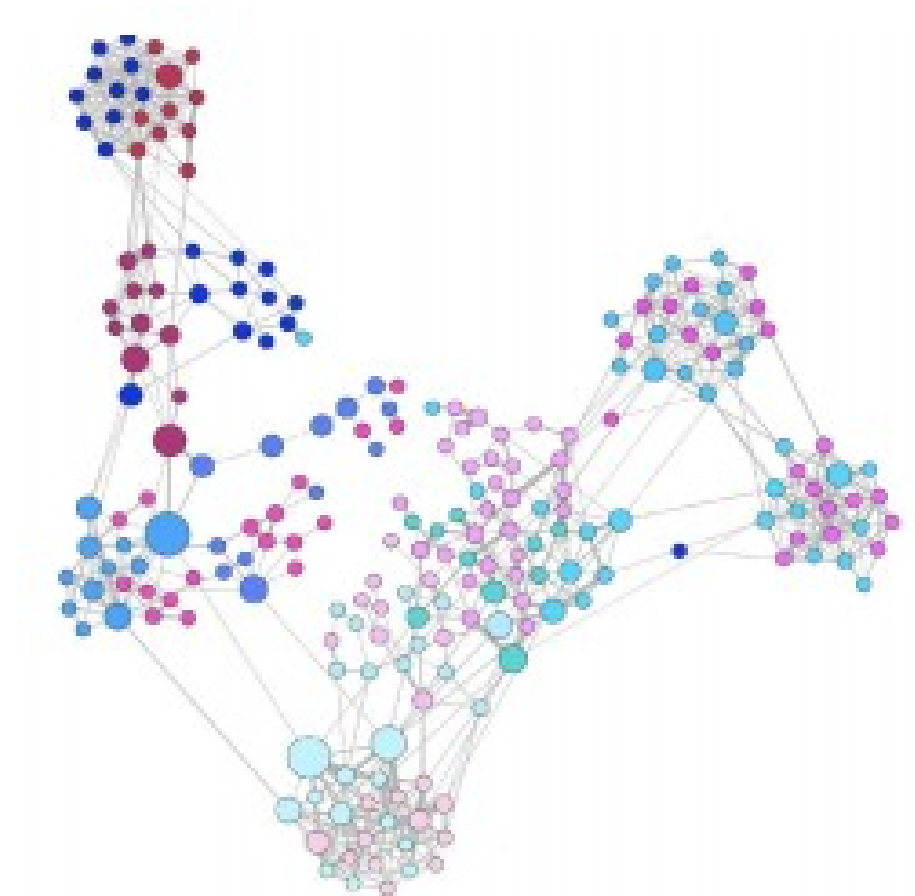
*Contacts in a network limits the paths a virus takes in a population*

# Contact networks

Contacts constrained  
by the network

Real degree instead of  
average

Differences on random and  
scale-free topologies



# IMMUNIZATION

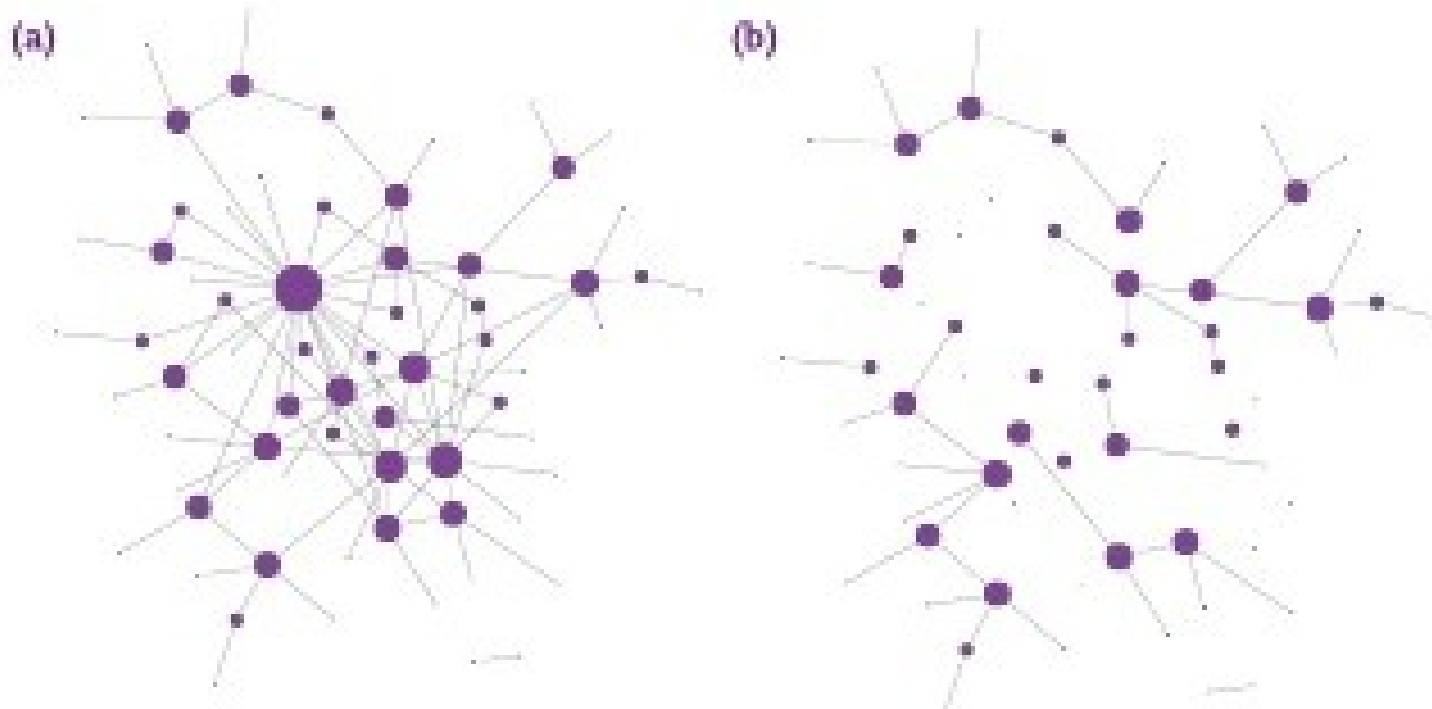
*Can we protect? Immunization strategies. Herd immunization*

# How to control a pandemic?

- Interventions to reduce transmission (masks, gloves,...)
- Contact-reducing interventions (quarantine)
- Vaccination: remove nodes from network

Who should be first vaccinated?

# Robustness and immunization



# Strategies

- Random immunization
  - Random selection
- Selective immunization
  - Random selection (G0)
  - Neighbors of G0 (G1)
  - illusion of majority  $\text{deg}(G1) > \text{deg}(G0)$ 
    - G1 vaccinated