An Introduction to Computational Argumentation Semantics (3/5)
Topic: Probabilistic Argumentation Frameworks

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Expanding Heathrow airport[1]

P: We should build a third runway at Heathrow because everyone will benefit from the increased capacity.

O: It is not true that everyone will benefit in the community.

P: Local residents won't have problems with traffic because we will increase public transport to the airport.

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Identify the attacks between the arguments.

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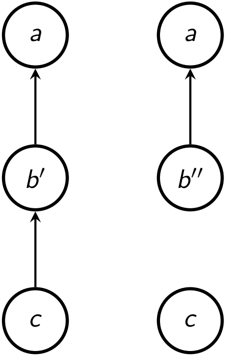
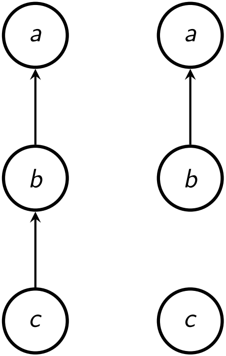


- *a* We should expand the airport as everyone would benefit.
- *b* Not everyone would benefit.
- *c* Increased road traffic will not hinder residents, as we will expand mass transit to the airport.

- b' Not everyone would benefit. Increased road traffic to the airport will hinder certain residents.
- b'' Not everyone would benefit. Some residents will suffer from increased air traffic noise.

Quantifying Uncertainty

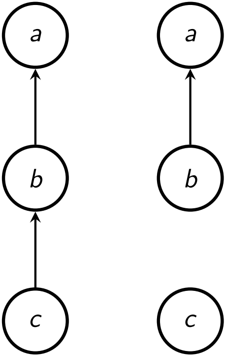


Probabilistic Argumentation Framework (PrAF)

A probabilistic argumentation framework expands an argumentation graph by associating each argument and attack with a likelihood value.

Argumentation graphs may be induced from a PrAF based on its probabilities.

Probabilities of extensions

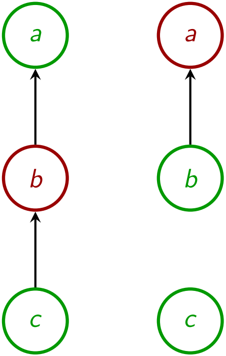


Probability of an Induced Graph

The probability of an induced graph is calculated as the joint probability of each argument and attack's absence or presence in the graph, assuming independence between probabilities.

The probability of an extension under a semantics is calculated by summing probabilities of induced graphs.

Probabilities of extensions



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Probabilistic Argumentation Frameworks[2]

Definition (PrAF)

A probabilistic argumentation framework (PrAF) is a quadruple

$$\mathsf{F} = \langle \mathcal{A}, P_{\mathcal{A}}, \mathcal{R}, P_{\mathcal{R}} \rangle,$$

- $\langle \mathcal{A}, \mathcal{R} \rangle$ is an argumentation graph,
- $P_{\mathcal{A}} : \mathcal{A} \to (0, 1]$ and
- $P_{\mathcal{R}} : \mathcal{R} \to (0, 1]$.

[2] Li, Oren and Norman, "Probabilistic Argumentation Frameworks."

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- $P_{\mathcal{A}}(a)$ – probability that a appears in the graph
- $P_{\mathcal{R}}((a,b))$ – conditional probability that the attack (a,b) appears in the graph, given that both a and b appear

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Calculating probabilities of extensions (1)

Definition (Induced Graph)

An argumentation graph $\mathsf{G}' = \langle \mathcal{A}', \mathcal{R}' \rangle$ is *induced* from $\mathsf{F} = \langle \mathcal{A}, P_{\mathcal{A}}, \mathcal{R}, P_{\mathcal{R}} \rangle$ iff the following hold:

- $\mathcal{A}' \subseteq \mathcal{A}$
- $\mathcal{R}' \subseteq \mathcal{R} \cap (\mathcal{A}' \times \mathcal{A}')$
- for every $a \in \mathcal{A}$ such that $P_{\mathcal{A}}(a) = 1$, $a \in \mathcal{A}'$
- for every $(a, b) \in \mathcal{R}$ such that $P_{\mathcal{R}}((a, b)) = 1$ and $a, b \in \mathcal{A}'$, $(a, b) \in \mathcal{R}'$

$\mathtt{I}(\mathsf{F})$ – the set of all induced graphs from F.

Calculating probabilities of extensions (2)

Probability of an induced graph

$$P^I_{\mathsf{F}}(\mathsf{G}') = \prod_{a \in \mathcal{A}'} P_{\mathcal{A}}(a) \prod_{a \in \mathcal{A} \setminus \mathcal{A}'} (1 - P_{\mathcal{A}}(a)) \prod_{r \in \mathcal{R}'} P_{\mathcal{R}}(r) \prod_{r \in \mathcal{R}\downarrow_{\mathcal{A}'} \setminus \mathcal{R}'} (1 - P_{\mathcal{R}}(r))$$

where $\mathcal{R} \downarrow_{\mathcal{A}'} = \{(a, b) | a, b \in \mathcal{A}' \text{ and } (a, b) \in \mathcal{R}\}$.

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It can be shown that

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Probability of an extension

For $\mathcal{E} \subseteq \mathcal{A}$, and a semantics σ,

$$Prob^{\sigma}(\mathcal{E}) = \sum_{\mathsf{G}: \mathcal{E} \in \sigma(\mathsf{G})} P^I_{\mathsf{F}}(\mathsf{G}) = 1$$

What's next

Thursday: Ranking/gradual semantics – is a stronger than b?

Friday: Structured argumentation – how to construct arguments and identify the attacks between them?