An Introduction to Computational Argumentation
Semantics (4/5)
Topic: Gradual Semantics

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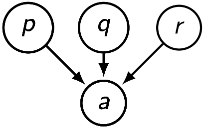
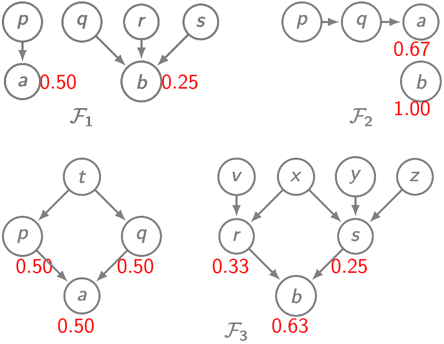
Gradual semantics (≈ Ranking-based semantics)



- Extension-based semantics compute jointly acceptable sets of arguments (extensions) – one successful attack has the same effect as several attacks
- For some applications, that is not desirable.
- Example: dialogues
  - a : *She is the best candidate for the position*
  - p : *She does not have enough teaching experience*
  - q : *She never published in this area*
  - r : *She is not fluent in English*
  - One attack does not have the same effect as several attacks
  - One attack does not completely destroy its target
- Gradual semantics
  - do not compute extensions
  - assign a unique score to each argument

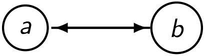
Some examples



- An example: h-categorizer (Besnard & Hunter, AIJ 2001)

$$\texttt{Deg}(a) = \frac{1}{1 + \sum_{b \mathcal{R} a} \texttt{Deg}(b)}$$

h-categorizer



- h-categorizer (Besnard & Hunter, AIJ 2001)

$$\mathtt{Deg}(a) = \frac{1}{1 + \sum_{b\mathcal{R}a} \mathtt{Deg}(b)}$$

- $\mathtt{Deg}(a) = ?$
- $\mathtt{Deg}(b) = ?$

- Solving a system of equations!

$$x = \frac{1}{1+y},\ y = \frac{1}{1+x}$$

- Does it always have a solution? Is it unique? How can we calculate it?

Two general settings for gradual semantics

Evaluation method setting [Cayrol and Lagasquie, JAIR 2005; Leite and Martins, IJCAI 2011, Amgoud and Doder AAMAS 2019]

How to define a gradual semantics in a general way, by a pair of functions (aggregation of strengths of attackers + effect of attacks on an argument).

Principle-based setting [Amgoud et al. IJCAI'17; Baroni, Rago, Toni IJAR 2019; Amgoud, Doder, Vesic, AIJ 2022]

Defines a semantics as a function that follows some high-level principles.

Note: Today, we will present both approaches to semantics for the class of weighted graphs

Weighted Graphs

Weighted Graph

$$G = \langle \mathcal{A}, w, \mathcal{R} \rangle$$

- $\mathcal{A}$ – arguments,
- $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ – attacks,
- $w : \mathcal{A} \rightarrow [0, 1]$ – basic weights of arguments

Weights:

- certainty degree of information
- reliability of the source
- aggregation of votes

Weighted *h*-categorizer

- This semantics extends *h*-categorizer
- Introduced by Amgoud et al. (IJCAI'17)

Definition

$$\mathtt{Deg}(a) = \frac{w(a)}{1 + \sum_{b\mathcal{R}a} \mathtt{Deg}(b)}$$

Evaluation method for the weighted *h*-categorizer:

- aggregation of strengths of attackers – $\sum$
- effect of attacks on the argument a – $f(x) = \frac{w(a)}{1+x}$

Social Abstract Argumentation Framework (Leite and Martins (IJCAI'11))

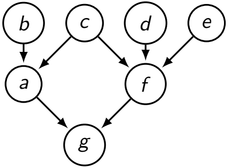
- Each argument receives positive and negative votes
- Votes of argument a are aggregated $\tau(a) = \frac{v^+}{v^+ + v^- + \epsilon}$
- Simple product semantics:
  $\mathtt{Deg}(a) = \tau(a) \cdot (1 - (\mathtt{Deg}(b_1) \oplus \cdots \oplus \mathtt{Deg}(b_n)))$, where
  - $b_1 \ldots b_n$ are the attackers of a
  - $x \oplus y = x + y - x \cdot y$

Principle-based setting

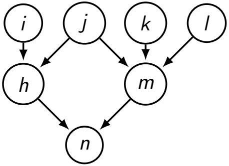
Why do we study principles?

- better understanding of semantics
- definition of reasonable semantics
- comparing semantics
- choosing suitable semantics for applications

Anonymity



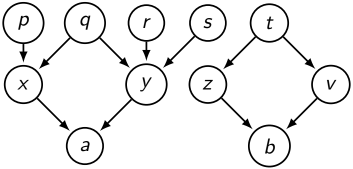
$\mathcal{F}_1$



$\mathcal{F}_2$

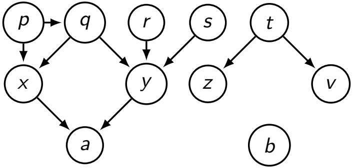
$$\frac{\begin{array}{c} w(g) = w(n) \\ w(a) = w(h) \\ \vdots \end{array}}{\begin{array}{c} \mathtt{Deg}(g) = \mathtt{Deg}(n) \\ \mathtt{Deg}(a) = \mathtt{Deg}(h) \\ \vdots \end{array}}$$

Independence



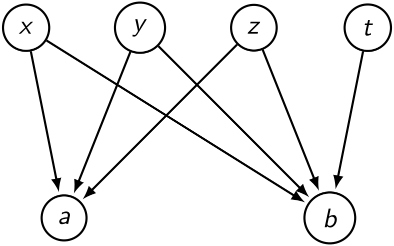
$\texttt{Deg}(a)$, $\texttt{Deg}(x)$, $\texttt{Deg}(y)$, ... stay the same

Directionality



no path from x to $y \Rightarrow \texttt{Deg}(y)$ does not change

Neutrality



$$\frac{w(a) = w(b) \quad \mathrm{Deg}(t) = 0}{\mathrm{Deg}(a) = \mathrm{Deg}(b)}$$

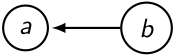
Equivalence

$$\frac{w(a) = w(b) \qquad \exists \text{ a bijection } f : \texttt{Att}(a) \to \texttt{Att}(b) \text{ s.t. } \forall x \in \texttt{Att}(a),\ \texttt{Deg}(x) = \texttt{Deg}(f(x))}{\texttt{Deg}(a) = \texttt{Deg}(b)}$$

Maximality

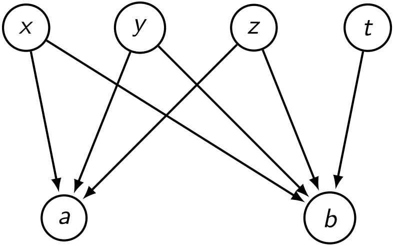
$$\frac{\texttt{Att}(a) = \emptyset}{\texttt{Deg}(a) = w(a)}$$

Weakening



$$\frac{w(a) > 0 \quad a \text{ is attacked by } b \text{ s.t. } \mathtt{Deg}(b) > 0}{\mathtt{Deg}(a) < w(a)}$$

Counting



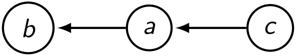
a has positive score

t has positive score

$w(a) = w(b)$

$\texttt{Deg}(a) > \texttt{Deg}(b)$

Weakening soundness



$$w(a) > 0$$
$$\mathtt{Deg}(a) < w(a)$$

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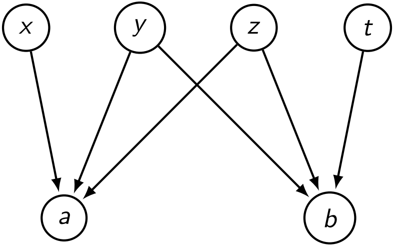
a is attacked by at least one argument c such that $\mathtt{Deg}(c) > 0$

Reinforcement



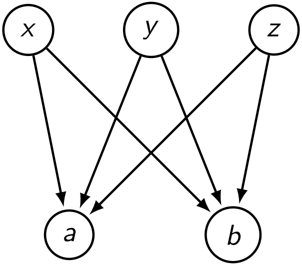
$$w(a) = w(b)$$
$$\texttt{Deg}(t) > \texttt{Deg}(x)$$
$$\texttt{Deg}(a) > 0 \text{ or } \texttt{Deg}(b) > 0$$

$$\texttt{Deg}(a) > \texttt{Deg}(b)$$

Resilience

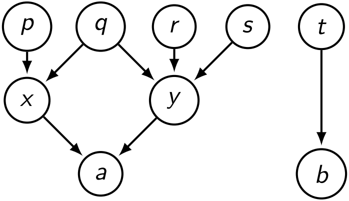
$$\frac{w(a) > 0}{\mathtt{Deg}(a) > 0}$$

Proportionality



$$\frac{\texttt{Att}(a) = \texttt{Att}(b) \quad w(a) > w(b) \quad \texttt{Deg}(a) > 0 \text{ or } \texttt{Deg}(b) > 0}{\texttt{Deg}(a) > \texttt{Deg}(b)}$$

Quality precedence / Quantity precedence / Compensation



Some results

Theorem

Let a semantics S satisfy Directionality, Independence, Maximality and Neutrality

- *Then, S satisfies Weakening soundness*
- *If S satisfies Reinforcement, then it satisfies both Counting and Weakening*

Proof

Suppose that S satisfies Directionality, Independence, Maximality and Neutrality and let us prove that it satisfies Weakening Soundness. Let $\mathrm{G} = \langle \mathcal{A}, w, \mathcal{R} \rangle$ be an argumentation graph and $a \in \mathcal{A}$. We prove by induction on $|\mathtt{Att}(a)|$ that if for every $b \in \mathtt{Att}_{\mathrm{G}}(a)$ we have that $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(b) = 0$ then $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a) = w(a)$.
Base. In the case when $|\mathtt{Att}_{\mathrm{G}}(a)| = 0$, Maximality implies that $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a) = w(a)$.
Step. Let the inductive hypothesis hold for all $k < n$ and suppose that $|\mathtt{Att}_{\mathrm{G}}(a)| = n$ and that all the attackers of a have degree 0. Let x be an arbitrary attacker of a. Denote $S = \mathtt{Att}_{\mathrm{G}}(a) \setminus \{x\}$. Let $\mathrm{G}' = \langle \mathcal{A}', w', \mathcal{R}' \rangle$ be such that $\mathcal{A}' = \mathcal{A} \cup \{y\}$ where y is a fresh argument (i.e. $y \notin \mathcal{A}$), $w'(t) = w(t)$ for all $t \in \mathcal{A}$, $w'(y) = w(a)$, $\mathcal{R} = \mathcal{R}'$. By independence, the degrees of arguments are same in G as in G$'$. By applying $n-1$ times directionality we conclude that the degrees of all arguments except y stay the same if we add the following set of attacks: $\{(z, y) \mid z \in S\}$. By inductive hypothesis, y's degree is identical to its weight. Thus, by Neutrality, the degree of a is also equal to its weight. By induction, we conclude that if for every $b \in \mathtt{Att}(a)$ we have that $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(b) = 0$ then $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a) = w(a)$. Weakening Soundness now follows from the previous fact by contraposition.

Exercise

Are the principles satisfied by h-categorizer?

BONUS PART: Linking the two settings

- Some more words about Evaluation method setting (EMS)
- Question: can we link EMS with principles?
- We answer the question for the extended framework (attacks are also weighted):

$$G = \langle \mathcal{A}, w, \mathcal{R}, \pi \rangle$$

  - $\mathcal{A}$ – arguments,
  - $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ – attacks,
  - $w : \mathcal{A} \to [0, 1]$ – basic weights of arguments
  - $\pi : \mathcal{R} \to [0, 1]$ - weights of attacks

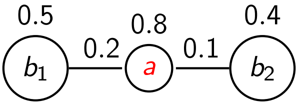
- Principles are extended in a straightforward way (Amgoud and Doder, AAMAS 2019)

Evaluation method setting

Evaluation Method

$$M = \langle f, g, h \rangle$$

- h calculates the *strength of one attack*
- g evaluates how strongly an argument is attacked.
- f returns the strength of an argument, using the value returned by g



- $\alpha_1 = h(0.5, 0.2), \quad \alpha_2 = h(0.4, 0.1)$
- $\gamma_a = g(\alpha_1, \alpha_2)$ – strength of attacks toward a
- $f(0.8, \gamma_a)$ – final strength of a

Evaluation method setting

Evaluation Method

$$\mathrm{M} = \langle f, g, h \rangle$$

- h calculates the *strength of one attack*
- g evaluates how strongly an argument is attacked.
- f returns the strength of an argument, using the value returned by g

$f_h(x_1, x_2) = x_1(1 - x_2)$	$g_{sum}(x_1, \ldots, x_n) = \sum_{i=1}^{n} x_i$	$h_{prod}(x_1, x_2) = x_1 x_2$
$f_{exp}(x_1, x_2) = x_1 e^{-x_2}$	$g_{sum,\alpha} = (\sum_{i=1}^{n} (x_i)^{\alpha})^{\frac{1}{\alpha}}$	$h_{prod,\alpha}(x_1, x_2) = x_1^{\alpha} x_2$
$f_{frac}(x_1, x_2) = \frac{x_1}{1+x_2}$	$g_{max} = \max\{x_i\}$	$h_{min}(x_1, x_2) = \min\{x_i$
$f_{min} = \min\{x_1, 1 - x_2\}$	$g_{psum} = x_1 \oplus \cdots \oplus x_n$, $x_1 \oplus x_2 = x_1 + x_2 - x_1 x_2$	$h_{Ham}(x_1, x_2) = \frac{x_1 x_2}{x_1 + x_2 -}$

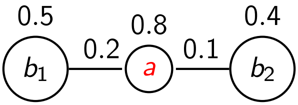
Gradual semantics via Evaluation methods

Definition

Let $\mathrm{M} = \langle f, g, h \rangle$,$\mathrm{G} = \langle \mathcal{A}, w, \mathcal{R}, \pi \rangle \in$, $a \in \mathcal{A}$.

$$\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a) = f(w(a), g(h(\pi((b_1, a)), \mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(b_1)), \ldots, h(\pi((b_n, a)), \mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(b_n))))$$

where $\{b_1, \ldots, b_n\} = \mathtt{Att}_{\mathrm{G}}(a)$.



- $\alpha_1 = h(0.5, 0.2), \quad \alpha_2 = h(0.4, 0.1)$
- $\gamma_a = g(\alpha_1, \alpha_2)$ – strength of attacks toward a
- $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a) = f(0.8, g(\alpha_1, \alpha_2))$= $f(0.8, g(h(0.5, 0.2), h(0.4, 0.1))$

Gradual semantics via Evaluation methods - questions

Definition

Let $\mathrm{M} = \langle f, g, h \rangle$,$\mathrm{G} = \langle \mathcal{A}, w, \mathcal{R}, \pi \rangle \in$, $a \in \mathcal{A}$.

$$\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a) = f(w(a), g(h(\pi((b_1, a)), \mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(b_1)), \ldots, h(\pi((b_n, a)), \mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(b_n))))$$

where $\{b_1, \ldots, b_n\} = \mathtt{Att}_{\mathrm{G}}(a)$.

- Degree $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a)$ depends on the degrees of attackers of a
- Does $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}(a)$ exist for every weighted graph?
- Easy case: no cycles $\Rightarrow$ $\mathtt{Deg}^{\mathrm{S}}_{\mathrm{G}}$ exists and is unique
- But in general case?

Intuitive constraints on f, g, h

Definition (Well-Behaved EM)

An evaluation method $\mathrm{M} = \langle f, g, h \rangle$ is well-behaved iff the following holds:

- f is increasing in the first variable,
- f is decreasing in the second variable whenever the first variable is not equal to 0,
- $f(x, 0) = x$, $f(0, x) = 0$.
- $g() = 0$, $g(x) = x$,
- $g(x_1, \ldots, x_n) = g(x_1, \ldots, x_n, 0)$, and
- $g(x_1, \ldots, x_n, y) \leq g(x_1, \ldots, x_n, z)$ if $y \leq z$,
- g is commutative,
- $h(0, x) = 0$,
- $h(1, x) = x$, $h(x, y) > 0$ whenever $xy > 0$, and
- h is non-decreasing in both components.

Well-Behaved EM $\Rightarrow$ principles

Theorem

If an EM $\mathrm{M} = \langle f, g, h \rangle$

- *is well-behaved*
- *defines a unique semantics*

then the semantics satisfies all the principles.

A class of evaluation methods for weighted graphs M^*

Definition (M^*)

M^* – the set of all well-behaved EMs $M = \langle f, g, h \rangle$ such that:

- $\lim_{x_2 \to x_0} f(x_1, x_2) = f(x_1, x_0)$, $\forall x_0 \neq 0$.
- $\lim_{x \to x_0} g(x_1, \ldots, x_n, x) = g(x_1, \ldots, x_n, x_0)$, $\forall x_0 \neq 0$.
- h is continuous on the second variable
- $\lambda f(x_1, \lambda x_2) < f(x_1, x_2)$, $\forall \lambda < 1$.
- $g(h(y_1, \lambda x_1), \ldots, h(y_n, \lambda x_n)) \geq \lambda g(h(y_1, x_1), \ldots, h(y_n, x_n))$, $\forall \lambda \in [0, 1]$.

Results (1)

Theorem

Every EM form M^* defines a unique semantics.

In addition, every semantics form the class can be effectively calculated:

Theorem

Let $\mathrm{M} = \langle f, g, h\rangle \in \mathrm{M}^*$, $\mathrm{S} = \mathrm{S}(\mathrm{M})$, *and* $\in$. *For every* $a \in \mathcal{A}$, *we define the sequence* $\{s(a)^{(n)}\}_{n=1}^{+\infty}$ *in the following way:*

- $s(a)^{(1)} = w(a)$,
- $s(a)^{(n+1)} = f(w(a), g(h(\pi((a_1, a)), s(a_1)^{(n)}), \ldots, h(\pi((a_k, a)), s(a_k)^{(n)})))$, *where* $\{a_1, \ldots a_k\} = \mathtt{Att}(a)$.

Then, for every $a \in \mathcal{A}$:

- $\{s(a)^{(n)}\}_{n=1}^{+\infty}$ *converges, and*
- $\lim\limits_{n \to +\infty} s(a)^{(n)} = \mathtt{Deg}_{\mathrm{G}}^{\mathrm{S}}(a)$.

Results (2)

Theorem

The class M^* generalizes the following semantics form the literature, and provides novel semantics (eg. $\mathtt{Deg}_{\mathsf{G}}(a) = w(a)\dot{e}^{-\max_{b\mathcal{R}a} \pi((b,a))\mathtt{Deg}_{\mathsf{G}}(b)}$*).*

Semantics	Formal definition
h-Categorizer [BH, AIJ-01]	$\mathtt{Deg}^h_{\mathsf{G}}(a) = \frac{1}{1+\sum_{b\mathcal{R}a} \mathtt{Deg}^h_{\mathsf{G}}(b)}$
Comp-based [ABDV, KR-16]	$s^{\alpha-\mathrm{BBS}}_{\mathsf{G}}(a) = 1 + \left(\sum_{b\mathcal{R}a} \frac{1}{(s(b))^\alpha}\right)^{1/\alpha}$
W. *h*-Cat. [ABDV, IJCAI-17]	$\mathtt{Deg}^(_{\mathsf{G}}a) = \frac{w(a)}{1+\sum_{b\mathcal{R}a} \mathtt{Deg}^(_{\mathsf{G}}b)}$
W.Max-based [ABDV, IJCAI-17]	$\mathtt{Deg}^(_{\mathsf{G}}a) = \frac{w(a)}{1+\max_{b\mathcal{R}a} \mathtt{Deg}^(_{\mathsf{G}}b)}$
W.Card-based [ABDV, IJCAI-17]	$\mathtt{Deg}^{\mathrm{Cbs}}_{\mathsf{G}}(a) = \frac{w(a)}{1+\lvert\mathtt{AttF}_{\mathsf{G}}(a)\rvert+\frac{\sum_{b\in\mathtt{AttF}_{\mathsf{G}}(a)} \mathtt{Deg}^{\mathrm{Cbs}}_{\mathsf{G}}(b)}{\lvert\mathtt{AttF}_{\mathsf{G}}(a)\rvert}}$