Logic-Based Explainable Artificial Intelligence

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ICREA, Univ. Lleida, Catalunya, Spain

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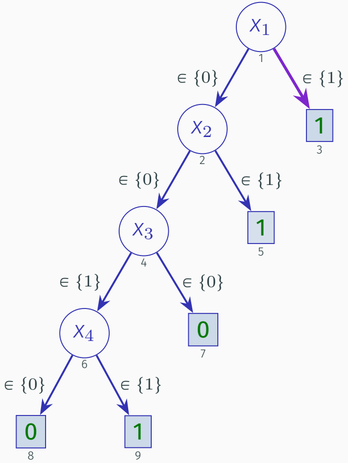
Lecture 04

Recapitulate third lecture

- Logic encoding for explaining DLs
  - And status of (in)tractability in logic-based XAI
- Query: enumeration of explanations
- Query: feature necessity, AXp & CXp
- Query: feature relevancy

Recap example

- Instance $(\mathbf{v}, c) = ((0,0,0,0),0)$



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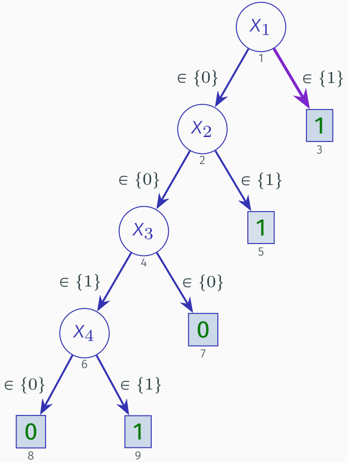

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- Instance $(\mathbf{v}, c) = ((0,0,0,0),0)$
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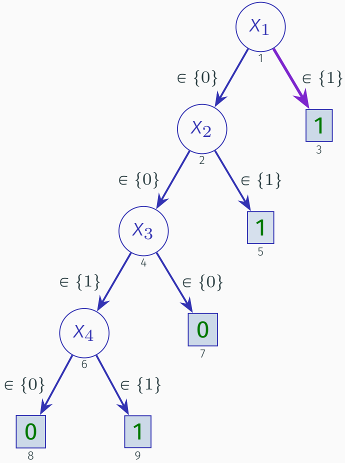

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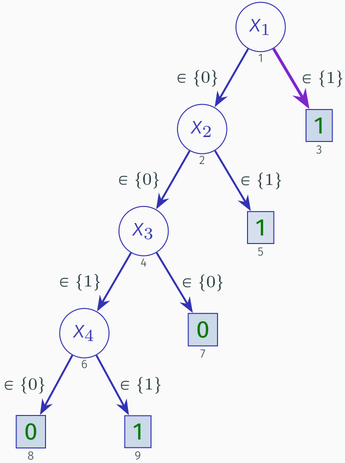

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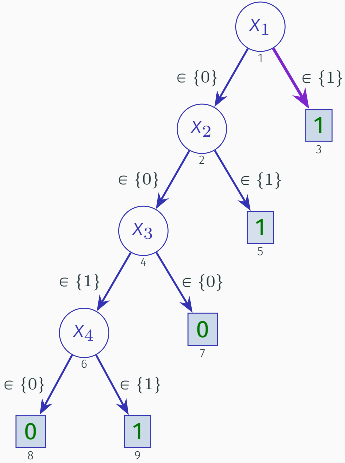

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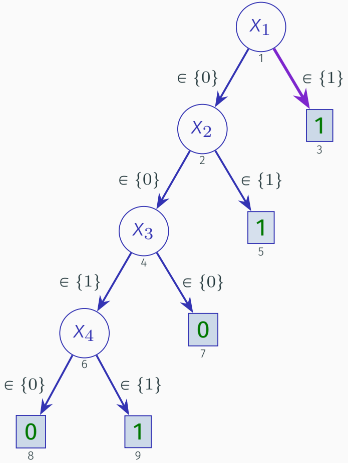

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  - Does there exist u_3, such that $\kappa(0,0,u_3,0) \neq \kappa(0,0,0,0)$?
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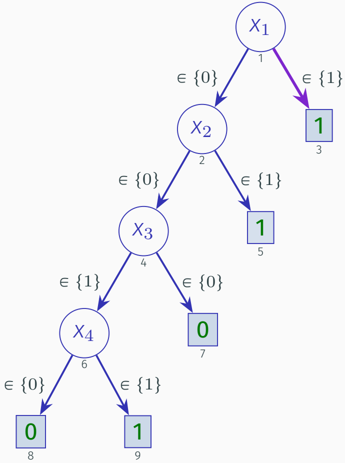

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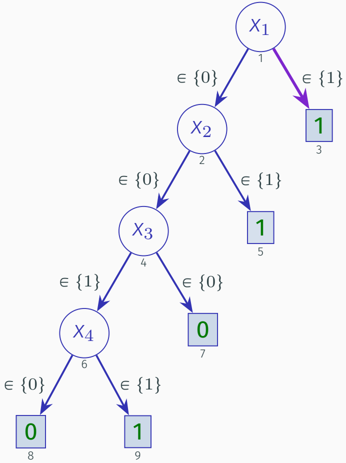
- Are there CXp-necessary features?
  - No! There are no singleton AXps



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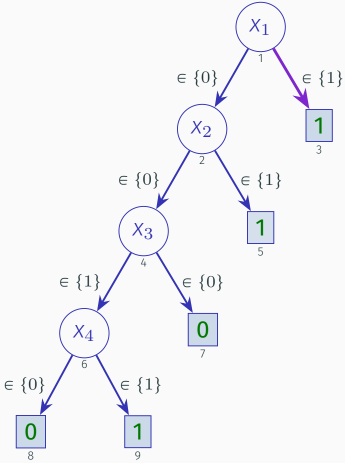

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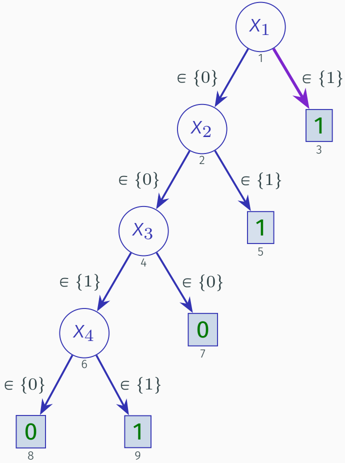
- Are there CXp-necessary features?
  - No! There are no singleton AXps
- Confirmation:
  - CXps: $\{\{1\},\{2\},\{3,4\}\}$ (2 is also AXp-necessary)
  - AXps: $\{\{1,2,3\},\{1,2,4\}\}$



Recap example – a different instance

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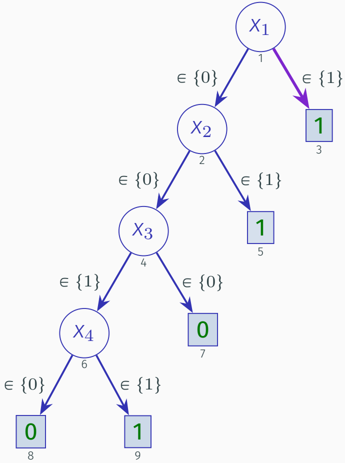

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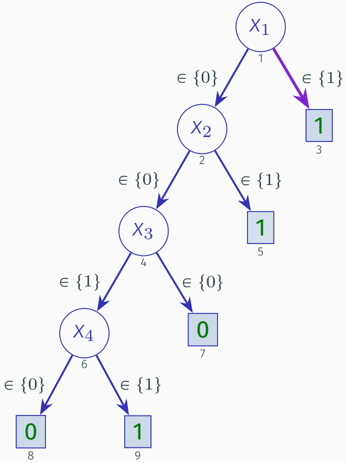

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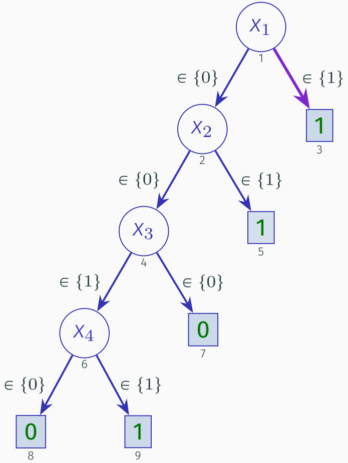

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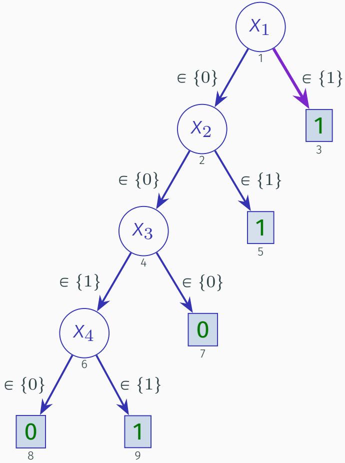

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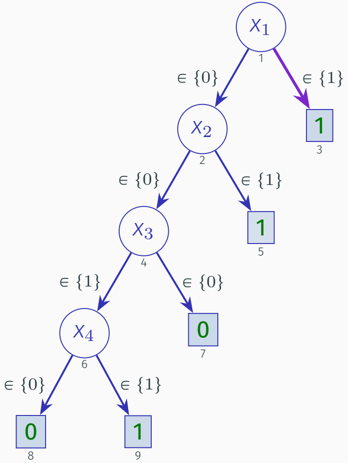

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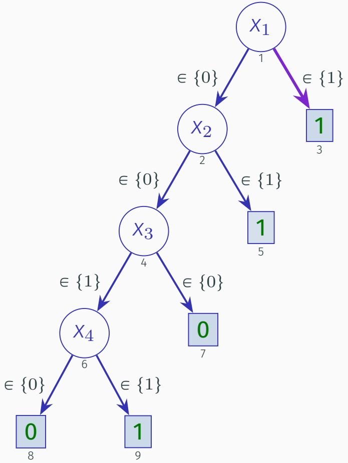

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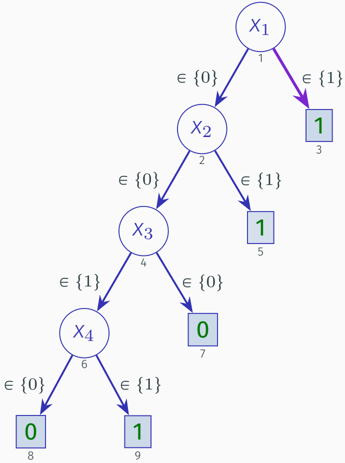

Another example – feature necessity & relevancy

- Classifier: $\mathcal{F} = \{1,2,3,4,5\}$; $\mathcal{D}_i = \{0,1\}$, $i = 1,\ldots,5$; $\mathcal{K} = \{0,1\}$

$$\kappa(x_1,x_2,x_3,x_4,x_5) \quad := \quad \begin{cases} 1 & \text{IF } (10x_1 + 5x_2 + 5x_3 + 2x_4 + x_5 \geqslant 15) \\ 0 & \text{otherwise} \end{cases}$$

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Q: What can we do if human decision maker finds computed AXp/CXp to be unsatisfactory?

- Partially enumerate AXps/CXps, exploiting bias in enumeration

Plan for this course

- Lecture 01 – units:
  - #01: Foundations
- Lecture 02 – units:
  - #02: Principles of symbolic XAI – feature selection
  - #03: Tractability in symbolic XAI (& myth of interpretability)
- Lecture 03 – units:
  - #04: Intractability in symbolic XAI (& myth of model-agnostic XAI)
  - #05: Explainability queries
- Lecture 04 – units:
  - #06: Advanced topics
- Lecture 05 – units:
  - #07: Principles of symbolic XAI – feature attribution (& myth of Shapley values in XAI)
  - #08: Conclusions & research directions

Detour: Monotonic Classification & Voting Power

Monotonically increasing boolean classifiers

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- Monotonic classifier $\mathcal{M} = (\mathcal{F}, \mathbb{F}, \mathcal{K}, \kappa)$, such that each $\mathcal{D}_i = \{0, 1\}$ and $\mathcal{K} = \{0, 1\}$ are ordered (i.e. $0 < 1$), and
  - $\kappa(\mathbf{1}) = 1$;
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- Let $\mathbf{v}_1, \mathbf{v}_2 \in \mathbb{F}$ be such that $\kappa(\mathbf{v}_1) = \kappa(\mathbf{v}_2) = 1$, and $\mathbf{v}_1 \leqslant \mathbf{v}_2$
  Define the explanation problems:
  - $\mathcal{E}_1 = (\mathcal{M}, (\mathbf{v}_1, 1))$
  - $\mathcal{E}_2 = (\mathcal{M}, (\mathbf{v}_2, 1))$
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- Then,
  - If WAXp$(\mathcal{S}; \mathcal{E}_1)$ holds, then WAXp$(\mathcal{S}; \mathcal{E}_2)$ holds; in particular:
  - $\mathbb{A}(\mathcal{E}_{\mathbb{1}})$ contains **all** the AXps of **any** instance of the form $(\mathbf{v}_r, 1)$
    - Why?
      - Pick any explanation problem $\mathcal{E}_r$ with instance $(\mathbf{v}_r, 1)$
      - Start from $\mathbb{1} = (1, 1, \ldots, 1)$
      - Remove features that take value 0 in $\mathbf{v}_r$; we still have an WAXp
      - Then compute any AXp starting from features taking value 1 in $\mathbf{v}_r$

      $\therefore$ **Suffices to find explanations for $\mathcal{E}_{\mathbb{1}}$** (or alternatively, the global explanations for prediction 1)

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    - AXp: $\{2, 3, 4, 5\}$; Q: Is feature 6 relevant?

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  - Problem: find a measure of importance of each voter !
    - I.e. measure the a priori voting power of each voter

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- **Q**: What should be the voting power of Luxembourg?

An example – EEC (EU) members voting power in 1958

Coutry	Acronym	# Votes
France	F	4
Germany	D	4
Italy	I	4
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Luxembourg	L	1
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- **Q**: What should be the voting power of Luxembourg?
- Can Luxembourg (L) ***matter*** for some winning coalition?
- Perhaps surprisingly, answer is **No**!
  - In 1958, Luxembourg was a **dummy** voter/player

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- The corresponding classifier is:

$$\kappa(x_1, x_2, x_3, x_4, x_5, x_6) \quad := \quad \begin{cases} 1 & \text{IF } (4x_1 + 4x_2 + 4x_3 + 2x_4 + 2x_5 + x_6 \geqslant 12) \\ 0 & \text{otherwise} \end{cases}$$

which we have seen before! E.g. $\{2, 3, 4, 5\}$ is an AXp & feature 6 (L) is irrelevant

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- Homework:
  - Create your own weighted voting games;
  - Compute the sets of AXps and CXps; and
  - Assess the importance of features and how they compare to each other

Unit #06

Advanced Topics

General definition of prediction sufficiency

- Instance $(\mathbf{v}, c)$
- Let $\mathcal{S} \subseteq \mathcal{F}$:
  - Recall,

$$\Upsilon(\mathcal{S};\mathbf{v}) = \{\mathbf{x} \in \mathbb{F} \mid \mathbf{x}_{\mathcal{S}} = \mathbf{v}_{\mathcal{S}}\}$$

- $\mathcal{S} \subseteq \mathcal{F}$ suffices for prediction c if:

$$\forall(\mathbf{x} \in \mathbb{F}).(\mathbf{x} \in \Upsilon(\mathcal{S};\mathbf{v})) \rightarrow (\sigma(\mathbf{x}))$$

  - **Obs:** a WAXp is just one possible example
- But there are other ways to study prediction sufficiency:
  - One can envision defining other sets of points Γ, parameterized by $\mathcal{E} = (\mathcal{M}, (\mathbf{v}, c))$; $\mathcal{S} \subseteq \mathcal{F}$ suffices for prediction c if:

$$\forall(\mathbf{x} \in \mathbb{F}).(\mathbf{x} \in \Gamma(\mathcal{S};\mathcal{E})) \rightarrow (\sigma(\mathbf{x}))$$

    - And one can also envision generalizations of σ!

Outline – Unit #06

[IISM24]

- Recall:

$$\text{WAXp}(\mathcal{X}) \quad := \quad \forall(\mathbf{x} \in \mathbb{F}).\bigwedge\nolimits_{j\in\mathcal{X}}(x_j = v_j) \rightarrow (\kappa(\mathbf{x}) = c)$$

  - For non-boolean features, use of $=$ may convey little information, e.g. with real-valued features, having $x_1 = 1.157$ does not help in understanding what values of feature 1 are also acceptable

Towards more expressive explanations – inflated explanations

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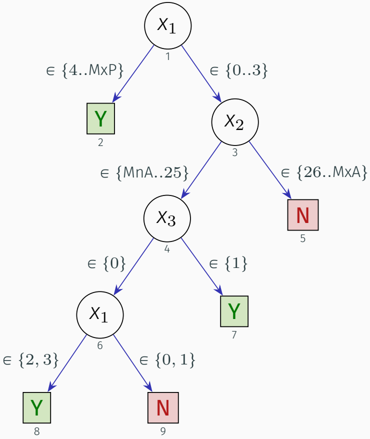
- Inflated explanations allow for more expressive literals, i.e. $=$ replaced with $\in$, and individual values replaced by ranges of values
  - Operational definition: Given an AXp, expand set of values of each feature, in some chosen order, such that the set of picked features remains unchanged

Inflated explanations – an example

[IIM22]



- Explanation for $((2, 20, 0), \text{Y})$? (Obs: $\text{MnA} = 18; \text{MxP} > 4$)

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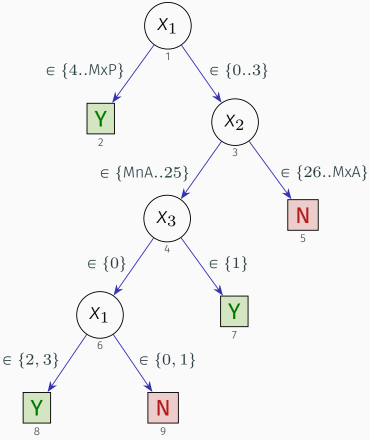

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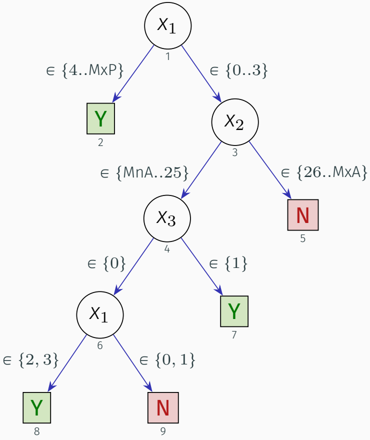

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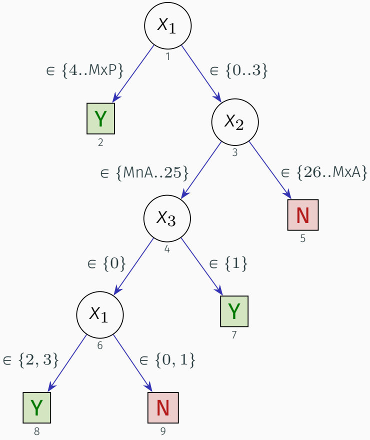

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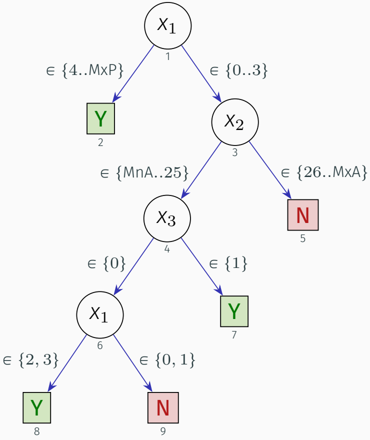

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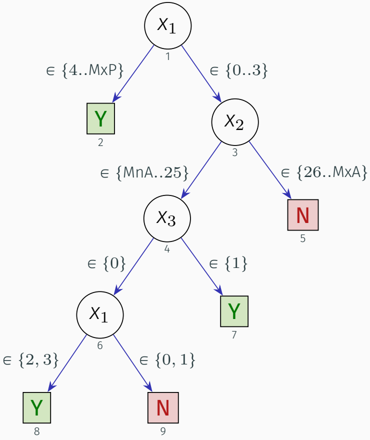

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- **Obs:** More complex alternative is to find AXp and expand domains simultaneously
    - This is conjectured to change the complexity class of finding one explanation

Outline – Unit #06

Probabilistic (formal) explanations

[WMHK21, IIN+22, IHI+22, ABOS22, IHI+23, IMM24]

- Explanation size is critical for human understanding [Mil56]
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- Explanation size is critical for human understanding [Mil56]
- Probabilistic explanations provide smaller explanations, by trading off rigor of explanation by explanation size
- Definition of weak probabilistic AXp $\mathcal{X} \subseteq \mathcal{F}$:

$$\text{WPAXp}(\mathcal{X}) \quad := \quad \mathbf{Pr}(\kappa(\mathbf{x}) = c) \,|\, \mathbf{x}_{\mathcal{X}} = \mathbf{v}_{\mathcal{X}}) \geqslant \delta$$

  - Obs: $\mathbf{x}_{\mathcal{X}} = \mathbf{v}_{\mathcal{X}}$ requires points $\mathbf{x} \in \mathbb{F}$ to match the values of $\mathbf{v}$ for the features dictated by $\mathcal{X}$
  - Obs: for $\delta = 1$ we obtain a WAXp

Definitions

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- Weak probabilistic AXp (WPAXp):

$$\text{WeakPAXp}(\mathcal{X}; \mathbb{F}, \kappa, \mathbf{v}, c, \delta) :=$$

$$\mathrm{Pr}_{\mathbf{x}}(\kappa(\mathbf{x}) = c \,|\, \mathbf{x}_{\mathcal{X}} = \mathbf{v}_{\mathcal{X}}) \geqslant \delta \; := \; \frac{|\{\mathbf{x} \in \mathbb{F} : \kappa(\mathbf{x}) = c \land (\mathbf{x}_{\mathcal{X}} = \mathbf{v}_{\mathcal{X}})\}|}{|\{\mathbf{x} \in \mathbb{F} : (\mathbf{x}_{\mathcal{X}} = \mathbf{v}_{\mathcal{X}})\}|} \geqslant \delta$$

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  - Recent approximate algorithms for complex ML models [IMM24]

Results for decision trees

Dataset	DT N	DT A	Path M	Path m	Path avg	δ	MinPAXp Length M	MinPAXp Length m	MinPAXp Length avg	MinPAXp Prec avg	MinPAXp Time avg	LmPAXp Length M	LmPAXp Length m	LmPAXp Length avg	LmPAXp Prec avg	LmPAXp $m_\subseteq$	LmPAXp Time avg	D	Anchor Length M	Anchor Length m	Anchor Length avg	Anchor Length $F_{\notin P}$	Anchor Prec avg	Anchor Time avg
						100	11	3	6.8	100	2.34	11	3	6.9	100	100	0.00	d	12	2	7.0	26.8	76.8	0.96
adult	1241	89	14	3	10.7	95	11	3	6.2	98.4	5.36	11	3	6.3	98.6	99.0	0.01	u	12	3	10.0	29.4	93.7	2.20
						90	11	2	5.6	94.6	4.64	11	2	5.8	95.2	96.4	0.01							
						100	12	1	4.4	100	0.35	12	1	4.4	100	100	0.00	d	31	1	4.8	58.1	32.9	3.10
dermatology	71	100	13	1	5.1	95	12	1	4.1	99.7	0.37	12	1	4.1	99.7	99.3	0.00	u	34	1	13.1	43.2	87.2	25.13
						90	11	1	4.0	98.8	0.35	11	1	4.0	98.8	100	0.00							
						100	12	2	4.8	100	0.93	12	2	4.9	100	100	0.00	d	36	2	7.9	44.8	69.4	1.94
kr-vs-kp	231	100	14	3	6.6	95	11	2	3.9	98.1	0.97	11	2	4.0	98.1	100	0.00	u	12	2	3.6	16.6	97.3	1.81
						90	10	2	3.2	95.4	0.92	10	2	3.3	95.4	99.0	0.00							
						100	12	4	8.2	100	16.06	11	4	8.2	100	100	0.00	d	16	3	13.2	43.1	71.3	12.22
letter	3261	93	14	4	11.8	95	12	4	8.0	99.6	18.28	11	4	8.0	99.5	100	0.00	u	16	3	13.7	47.3	66.3	10.15
						90	12	4	7.7	97.7	16.35	10	4	7.8	97.8	100	0.00							
						100	14	3	6.4	100	0.92	14	3	6.5	100	100	0.00	d	35	2	8.6	55.4	33.6	5.43
soybean	219	100	16	3	7.3	95	14	3	6.4	99.8	0.95	14	3	6.4	99.8	100	0.00	u	35	3	19.2	66.0	75.0	38.96
						90	14	3	6.1	98.1	0.94	14	3	6.1	98.2	98.5	0.00							
						0	12	3	7.4	100	1.23	12	3	7.5	100	100	0.01	d	38	2	6.3	65.3	63.3	24.12
spambase	141	99	14	3	8.5	95	9	1	3.7	96.1	2.16	9	1	3.8	96.5	100	0.01	u	57	3	28.0	86.2	65.3	834.70
						90	6	1	2.4	92.4	2.15	8	1	2.4	92.2	100	0.01							

Results for naive Bayes classifiers

Dataset	(#F	#I)	NBC	AXp		LmPAXp$_{\leqslant 9}$				LmPAXp$_{\leqslant 7}$				LmPAXp$_{\leqslant 4}$			
			A%	Length	δ	Length	Precision	W%	Time	Length	Precision	W%	Time	Length	Precision	W%	Time
adult	(13	200)	81.37	6.8± 1.2	98	6.8± 1.1	100± 0.0	100	0.003	6.3± 0.9	99.61± 0.6	96	0.023	4.8± 1.3	98.73± 0.5	48	0.059
					95	6.8± 1.1	99.99± 0.2	100	0.074	5.9± 1.0	98.87± 1.8	99	0.058	3.9± 1.0	96.93± 1.1	80	0.071
					93	6.8± 1.1	99.97± 0.4	100	0.104	5.7± 1.3	98.34± 2.6	100	0.086	3.4± 0.9	95.21± 1.6	90	0.093
					90	6.8± 1.1	99.95± 0.6	100	0.164	5.5± 1.4	97.86± 3.4	100	0.100	3.0± 0.8	93.46± 1.5	94	0.103
agaricus	(23	200)	95.41	10.3± 2.5	98	7.7± 2.7	99.12± 0.8	92	0.593	6.4± 3.0	98.75± 0.6	87	0.763	6.0± 3.1	98.67± 0.5	29	0.870
					95	6.9± 3.1	97.62± 2.1	95	0.954	5.3± 3.2	96.59± 1.6	92	1.273	4.8± 3.3	96.24± 1.2	55	1.217
					93	6.5± 3.1	96.65± 2.8	95	1.112	4.8± 3.1	95.38± 1.9	93	1.309	4.3± 3.1	94.92± 1.3	64	1.390
					90	5.9± 3.3	94.95± 4.1	96	1.332	4.0± 3.0	92.60± 2.8	95	1.598	3.6± 2.8	92.08± 1.7	76	1.830
chess	(37	200)	88.34	12.1± 3.7	98	8.1± 4.1	99.27± 0.6	64	0.383	5.9± 4.9	98.70± 0.4	64	0.454	5.7± 5.0	98.65± 0.4	46	0.457
					95	7.7± 3.8	98.51± 1.4	68	0.404	5.5± 4.4	97.90± 0.9	64	0.483	5.3± 4.5	97.85± 0.8	46	0.478
					93	7.3± 3.5	97.56± 2.4	68	0.419	5.0± 4.1	96.26± 2.2	64	0.485	4.8± 4.1	96.21± 2.1	64	0.493
					90	7.3± 3.5	97.29± 2.9	70	0.413	4.9± 4.0	95.99± 2.6	64	0.483	4.8± 4.0	95.93± 2.5	64	0.543
vote	(17	81)	89.66	5.3± 1.4	98	5.3± 1.4	100± 0.0	100	0.000	5.3± 1.3	99.95± 0.2	100	0.007	4.6± 1.1	99.60± 0.4	64	0.014
					95	5.3± 1.4	100± 0.0	100	0.000	5.3± 1.3	99.93± 0.3	100	0.008	4.1± 1.0	98.25± 1.7	64	0.018
					93	5.3± 1.4	100± 0.0	100	0.000	5.2± 1.3	99.78± 1.1	100	0.012	4.1± 0.9	98.10± 1.9	64	0.018
					90	5.3± 1.4	100± 0.0	100	0.000	5.2± 1.3	99.78± 1.1	100	0.012	4.0± 1.2	97.24± 3.1	64	0.022
kr-vs-kp	(37	200)	88.07	12.2± 3.9	98	7.8± 4.2	99.19± 0.5	64	0.387	6.5± 4.7	98.99± 0.4	64	0.427	6.1± 4.9	98.88± 0.3	43	0.457
					95	7.3± 3.9	98.29± 1.4	64	0.416	6.0± 4.3	97.89± 1.1	64	0.453	5.5± 4.5	97.79± 0.9	43	0.462
					93	6.9± 3.5	97.21± 2.5	69	0.422	5.6± 3.8	96.82± 2.2	64	0.448	5.2± 4.0	96.71± 2.1	43	0.468
					90	6.8± 3.5	96.65± 3.1	69	0.418	5.4± 3.8	95.69± 3.0	64	0.468	5.0± 4.0	95.59± 2.8	61	0.487
mushroom	(23	200)	95.51	10.7± 2.3	98	7.5± 2.4	98.99± 0.7	90	0.641	6.5± 2.6	98.74± 0.5	83	0.751	6.3± 2.7	98.70± 0.4	18	0.828
					95	6.5± 2.6	97.35± 1.8	96	1.011	5.1± 2.5	96.52± 1.0	90	1.130	5.0± 2.5	96.39± 0.8	54	1.113
					93	5.8± 2.8	95.77± 2.7	96	1.257	4.4± 2.5	94.67± 1.6	94	1.297	4.2± 2.4	94.48± 1.3	65	1.324

Results for decision diagrams

Dataset	#I	#F	OMDD		δ	MinPAXp					LmPAXp					
						Length			Prec	Time	Length			Prec	$m_\subseteq$	Time
			#N	A%		M	m	avg	avg	avg	M	m	avg	avg		avg
lending	100	9	1103	81.7	100	9	6	8.0	100	24.24	9	6	7.9	100	100	1.57
					95	9	5	7.8	99.7	21.48	9	6	7.8	99.8	100	1.49
					90	9	4	7.2	96	24.65	9	5	7.4	97.0	100	1.48
monk2	100	6	70	79.3	100	6	4	5.1	100	0.10	6	4	5.1	100	100	0.03
					95	6	4	5.1	100	0.09	6	4	5.1	100	100	0.03
					90	6	3	4.8	98.1	0.09	6	3	4.8	98.1	100	0.03
postoperative	74	8	109	80	100	8	4	6.1	100	0.26	8	4	6.2	100	100	0.04
					95	8	2	6.0	99.3	0.25	8	2	6.0	99.3	100	0.04
					90	8	2	5.3	95.9	0.23	8	2	5.4	96.6	94.6	0.04
tic_tac_toe	100	9	424	70.3	100	9	5	7.7	100	3.60	9	5	7.8	100	100	0.38
					95	9	5	7.5	99.5	3.24	9	5	7.7	99.6	99.0	0.38
					90	9	3	7.3	98.3	4.06	9	3	7.5	98.6	98.0	0.38
xd6	100	9	76	83.1	100	9	4	4.6	100	0.10	9	4	4.6	100	100	0.03
					95	9	3	3.8	97	0.09	9	3	3.8	97.0	99.0	0.03
					90	9	3	3.3	94.8	0.10	9	3	3.4	94.6	100	0.03

Remarks on LmPAXps

[IHI+23]

- LmPAXps ignore non-monotonicity, and so overapproximate PAXps
  - Theoretical guarantees, but may be reducible
- For DTs, computation of LmPAXps is in P
- Experimental results confirm LmPAXps match PAXps in most cases
- Recent results on approximating LmPAXps for RFs [IMM24]

Outline – Unit #06

Not all inputs may be possible – input constraints

[GR22, YIS[+]23]

- The (implicit) assumption that all inputs are possible is often unrealistic
  - I.e. it may be impossible for some points in feature space to be observed

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$$\forall(\mathbf{x} \in \mathbb{F}).\left[\bigwedge\nolimits_{j\in\mathcal{X}}(x_j = v_j) \land \mathcal{C}(\mathbf{x})\right] \rightarrow (\kappa(\mathbf{x}) = c)$$

$$\exists(\mathbf{x} \in \mathbb{F}).\left[\bigwedge\nolimits_{j\in\mathcal{X}}(x_j = v_j) \land \mathcal{C}(\mathbf{x})\right] \land (\kappa(\mathbf{x}) \neq c)$$

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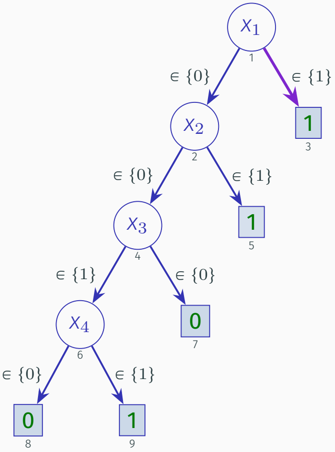
  - Compute AXps/CXps given new definitions
- Constrained AXps/CXps find other applications!

An example

- Instance: $((1,1,1,1),1)$

- Unconstrained AXps:



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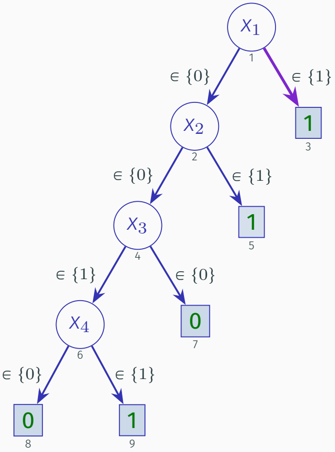

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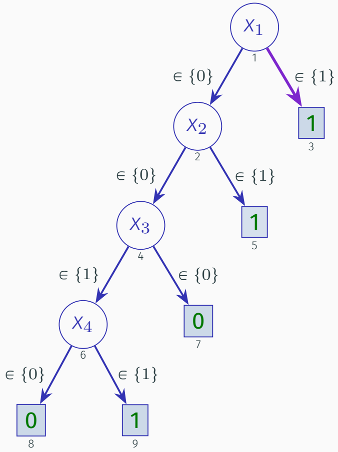

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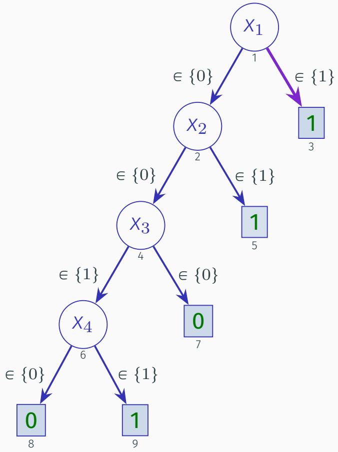

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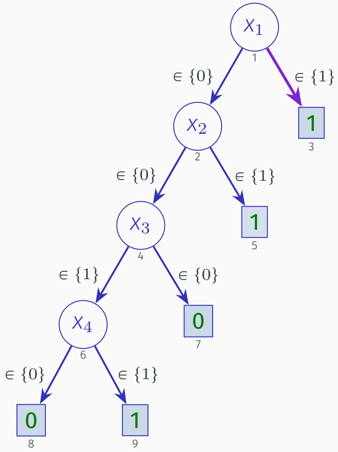

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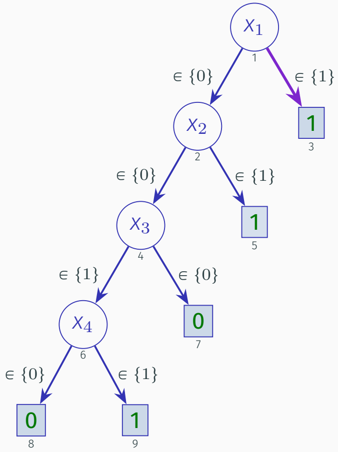

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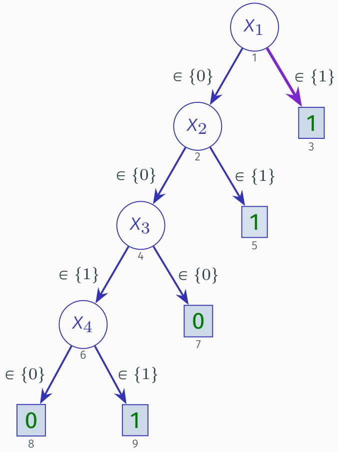

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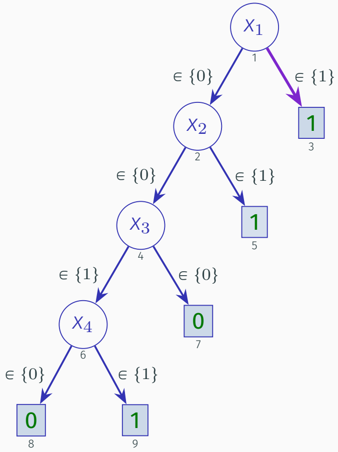

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Outline – Unit #06

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- Change definition of WAXp/WCXp to account for l_p distance to $\mathbf{v}$:

$$\forall(\mathbf{x} \in \mathbb{F}). \left[\bigwedge\nolimits_{j\in\mathcal{X}}(x_j = v_j) \land \left(\|\mathbf{x}-\mathbf{v}\|_{l_p} \leqslant \epsilon\right)\right] \rightarrow (\sigma(\mathbf{x}))$$

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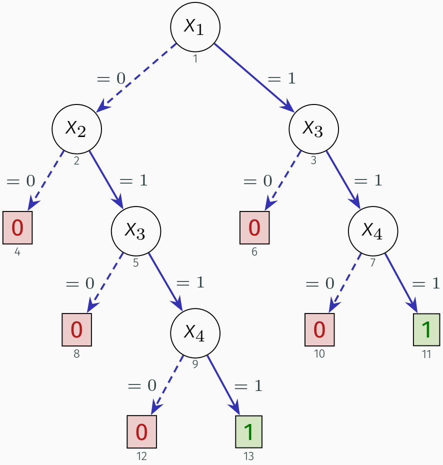
  - Norm l_p is arbitrary, e.g. Hamming, Manhattan, Euclidean, etc.
  - **Distance-restricted explanations**: $\mathfrak{d}$AXp/$\mathfrak{d}$CXp

An example – DT & instance $((1,1,1,1),1)$



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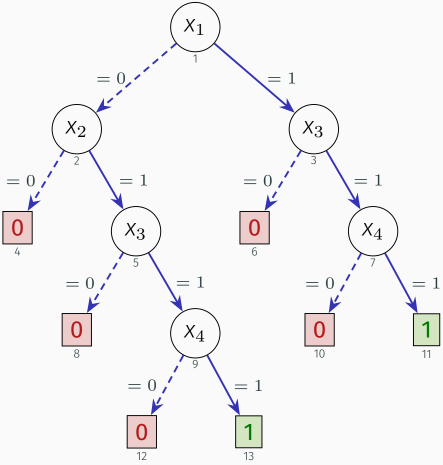

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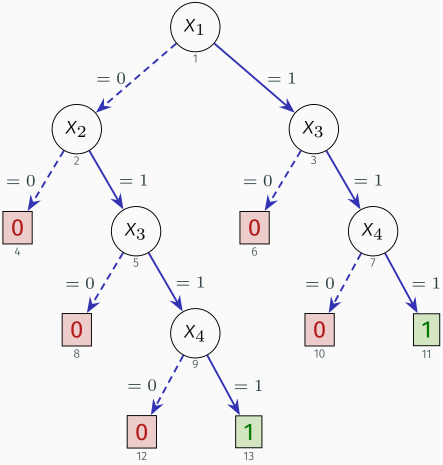

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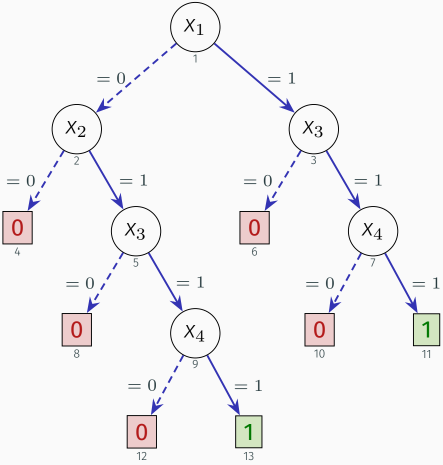

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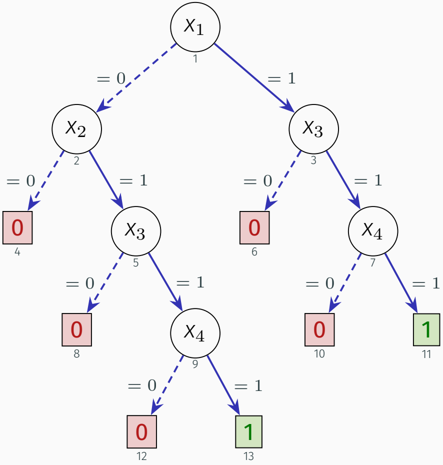

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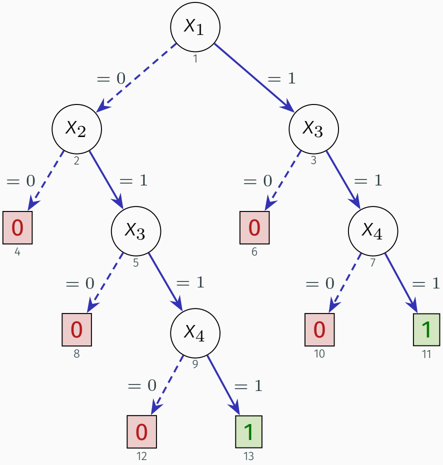

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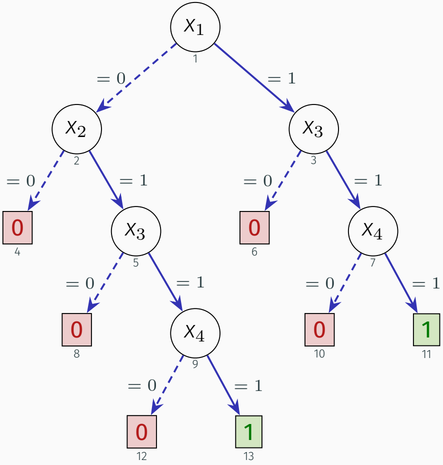

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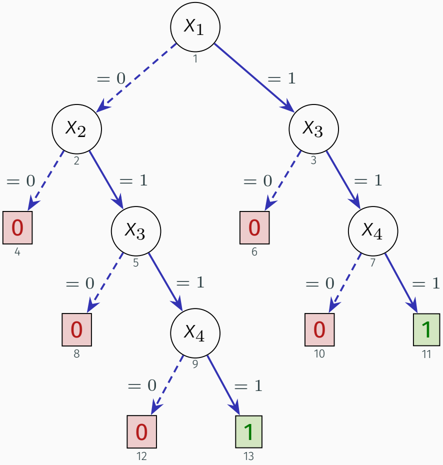

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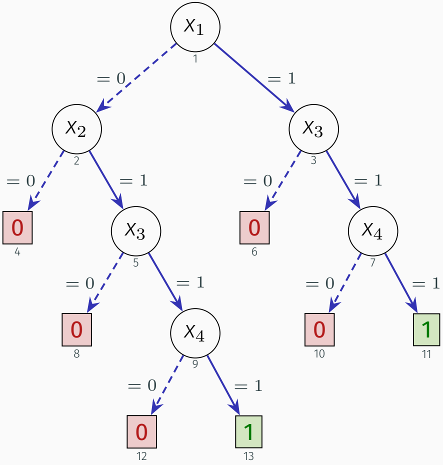

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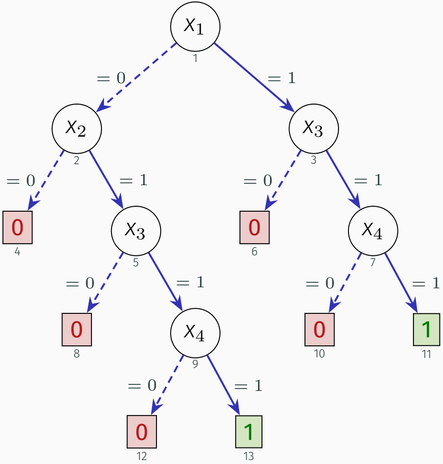
  - ∂CXps? $\{\{3\},\{4\}\}$



An example – DT & instance $((1,1,1,1),1)$

- Plain AXps/CXps:
  - AXps? $\{\{1,3,4\},\{2,3,4\}\}$
  - CXps? $\{\{1,2\},\{3\},\{4\}\}$
- Distance-restricted AXps/CXps, $\mathfrak{d}$AXp/$\mathfrak{d}$CXp, with Hamming distance (l_0) and $\epsilon = 1$:
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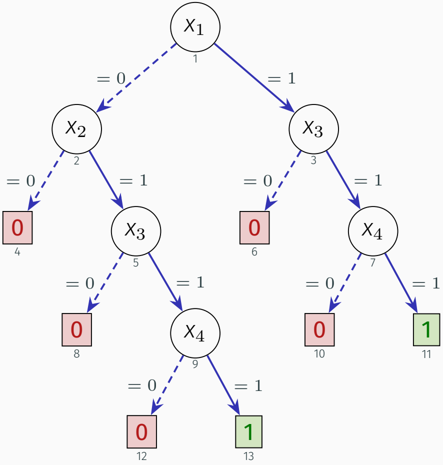
  - $\mathfrak{d}$AXps? $\{\{3,4\}\}$
  - $\mathfrak{d}$CXps? $\{\{3\},\{4\}\}$
- Given ϵ, larger adversarial examples are excluded



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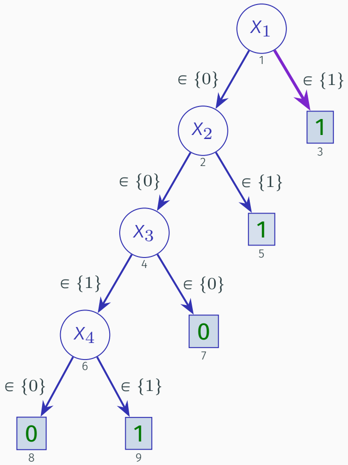

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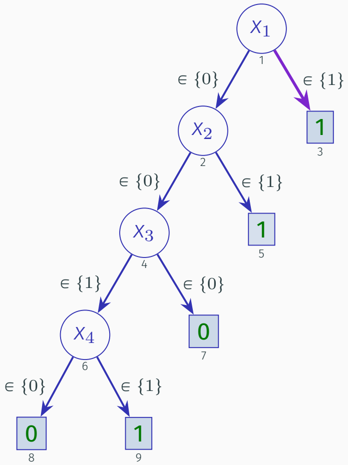

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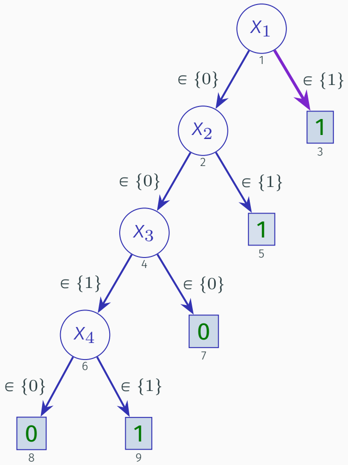

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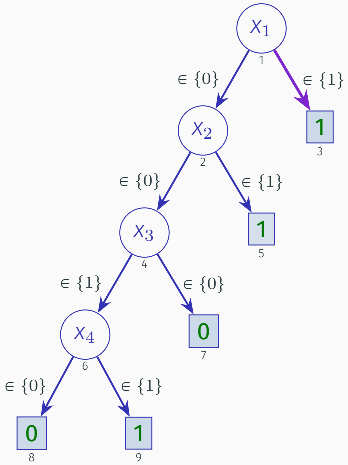

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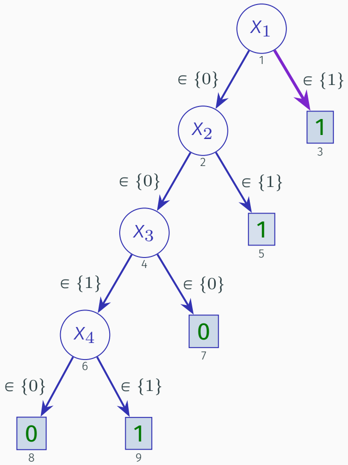

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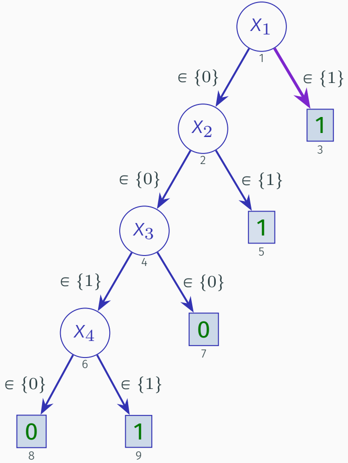

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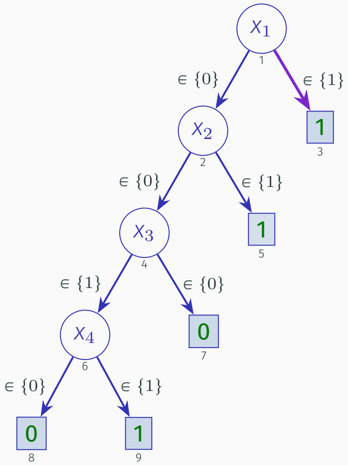

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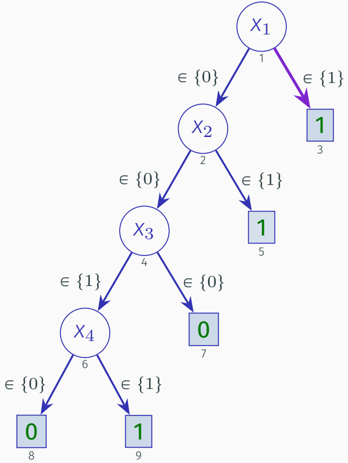

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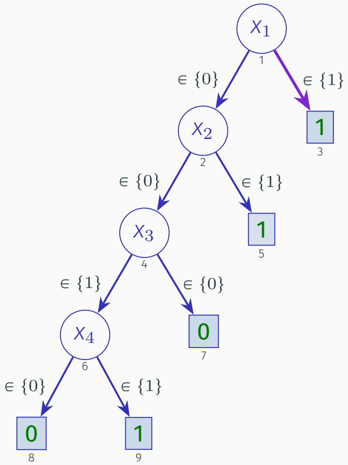

Relating explanations with adversarial examples

- Distance-restricted WAXps/WCXps:

$$\forall(\mathbf{x} \in \mathbb{F}). \left[\bigwedge_{j\in\mathcal{X}}(x_j = v_j) \wedge \left(\|\mathbf{x}-\mathbf{v}\|_{l_p} \leqslant \epsilon\right)\right] \rightarrow (\sigma(\mathbf{x}))$$

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- Clear scalability improvements for explaining NNs (see next) [HM23a, WWB23, IHM+24a, IHM+24b]

Basic algorithm

[HM23a]

Input: Arguments: ϵ; Parameters: $\mathcal{E}$, p
Output: One $\mathfrak{d}$AXp $\mathcal{S}$

```
function FindAXpDel(ε; ℰ, p)
    𝒮 ← ℱ                              ▷ Initially, no feature is allowed to change
    for i ∈ ℱ do                        ▷ Invariant: ∂WAXp(𝒮)
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        outc ← FindAdvEx(ε, 𝒮; ℰ, p)
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```

- **Obs:** Efficiency of logic-based XAI tracks efficiency of robustness tools
- **Limitation:** Running time grows with number of features

Results for NNs in 2023 (using Marabou [KHI+19])

DNN	points	|AXp|	#Calls	Time	#TO	|AXp|	#Calls	Time	#TO
				$\epsilon = 0.1$				$\epsilon = 0.05$	
ACASXU_1_5	#1	3	5	185.9	0	2	5	113.8	0
	#2	2	5	273.8	0	1	5	33.2	0
	#3	0	5	714.2	0	0	5	4.3	0
ACASXU_3_1	#1	0	5	2219.3	0	0	5	14.2	0
	#2	2	5	4263.5	1	0	5	1853.1	0
	#3	1	5	581.8	0	0	5	355.9	0
ACASXU_3_2	#1	3	5	13739.3	2	1	5	6890.1	1
	#2	3	5	226.4	0	2	5	125.1	0
	#3	2	5	1740.6	0	2	5	173.6	0
ACASXU_3_5	#1	4	5	43.6	0	2	5	59.4	0
	#2	3	5	5039.4	0	2	5	4303.8	1
	#3	2	5	5574.9	1	2	5	2660.3	0
ACASXU_3_6	#1	1	5	6225.0	1	0	5	51.0	0
	#2	3	5	4957.2	1	2	5	1897.3	0
	#3	1	5	196.1	0	1	5	919.2	0
ACASXU_3_7	#1	3	5	6256.2	0	4	5	26.9	0
	#2	4	5	311.3	0	1	5	6958.6	1
	#3	2	5	7756.5	1	1	5	7807.6	1
ACASXU_4_1	#1	2	5	12413.0	2	1	5	5090.5	1
	#2	1	5	5035.1	1	0	5	2335.6	0
	#3	4	5	1237.3	0	4	5	1143.4	0
ACASXU_4_2	#1	4	5	15.9	0	4	5	12.1	0
	#2	3	5	1507.6	0	1	5	111.3	0
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Scales to a few **hundred** neurons

Recent improvements

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  - Algorithm FindAXpDel is mostly sequential (see above)
  - Exploit parallelization for other algorithms, e.g. dichotomic search [IHM+24b]
- However, to decide whether $\mathcal{S}$ is an AXp, we can exploit parallelization:
  - Recall: $\text{AXp}(\mathcal{X}) := \text{WAXp}(\mathcal{X}) \wedge \forall(t \in \mathcal{X}).\neg\text{WAXp}(\mathcal{X} \setminus \{t\})$
  - Each $\neg$WAXp($\cdot$) (and also WAXp($\cdot$)) check can be run in parallel!
  - Do this opportunistically, i.e. when set $\mathcal{S}$ is expected to be AXp [IHM+24b]

More recent results (from 2024)...

[IHM+24a, IHM+24b]

Model	Deletion							SwiftXplain						
	avgC	nCalls	Len	Mn	Mx	avg	TO	avgC	nCalls	Len	FD%	Mn	Mx	avg
gtsrb-dense	0.06	1024	448	52.0	76.3	63.1	0	0.23	54	447	77.4	10.8	14.0	12.2
gtsrb-convSmall	0.06	1024	309	59.2	82.6	65.1	0	0.22	74	313	39.7	15.1	19.5	16.2
gtsrb-conv	—	—	—	—	—	—	100	96.49	45	174	33.2	3858.7	6427.7	4449.4
mnist-denseSmall	0.28	784	177	190.9	420.3	220.4	0	0.77	111	180	15.5	77.6	104.4	85.1
mnist-dense	0.19	784	231	138.1	179.9	150.6	0	0.75	183	229	11.5	130.1	145.5	136.8
mnist-convSmall	—	—	—	—	—	—	100	98.56	52	116	21.3	4115.2	6858.3	5132.8

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Largest for MNIST: **10142** neurons
Largest for GSTRB: **94308** neurons

Outline – Unit #06

Surrogate models in logic-based XAI

[BAMT21]

- Motivation:
  - Logic-based XAI does not yet scale for highly complex ML models
  - Surrogate models find many uses in ML, for approximating complex models

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- Explain the surrogate model
  - Compute rigorous explanation: plain AXp, probabilistic AXp,

Surrogate models in logic-based XAI

[BAMT21]

- Motivation:
  - Logic-based XAI does not yet scale for highly complex ML models
  - Surrogate models find many uses in ML, for approximating complex models
- Approach:
  - Train a surrogate model, e.g. DT, RF/TE, small(er) NN, etc.
  - Target high accuracy of surrogate model
- Explain the surrogate model
  - Compute rigorous explanation: plain AXp, probabilistic AXp,
- Report computed explanation as explanation for the complex ML model

Certified explainer (for monotonic classification)

[HM23c]

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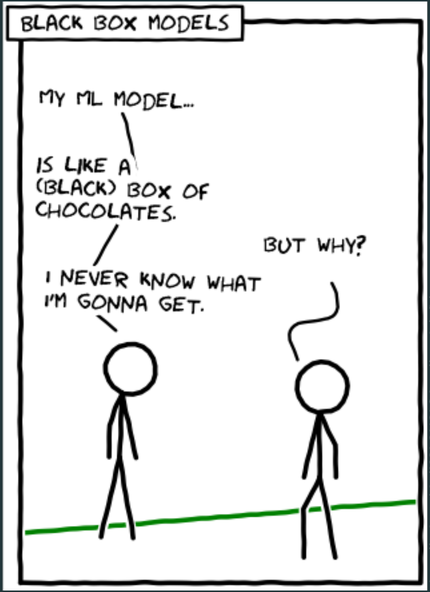
- Certification envisioned for **any** explainability algorithm

Plan for this course – light at the end of the tunnel...

- Lecture 01 – units:
  - #01: Foundations
- Lecture 02 – units:
  - #02: Principles of symbolic XAI – feature selection
  - #03: Tractability in symbolic XAI (& myth of interpretability)
- Lecture 03 – units:
  - #04: Intractability in symbolic XAI (& myth of model-agnostic XAI)
  - #05: Explainability queries
- Lecture 04 – units:
  - #06: Advanced topics
- Lecture 05 – units:
  - #07: Principles of symbolic XAI – feature attribution (& myth of Shapley values in XAI)
  - #08: Conclusions & research directions

Questions?



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