ESSAI 2024 course: Logic-based specification and verification of multi-agent systems

Lecture 4.1: Generalised Dining Philosophers games: Competitive dynamic resource allocation in MAS

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Main reference

Riccardo De Masellis, Valentin Goranko, Stefan Gruner, and Nils Timm. *Generalising the Dining Philosophers Problem: Competitive Dynamic Resource Allocation in Multi-agent Systems*,

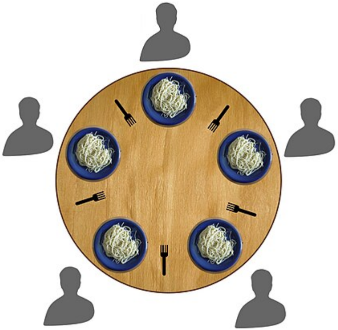
Proceedings of the 16th European Conference in Multi-agent Systems (EUMAS2018), Springer LNCS vol.11450, 2019, pp. 30-47.
https://link.springer.com/chapter/10.1007/978-3-030-14174-5_3

Generalised Dining Philosophers Games:

Informal introduction

Dijkstra's dining philosophers problem

Edsger Dijkstra, 1965:

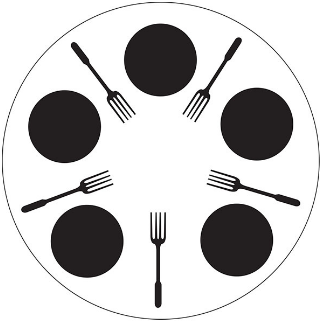


- Five philosophers dining with spaghetti at a round table.
- Five forks are available, as on the figure.
- Every philosopher either thinks or eats at any time instant.
- Every philosopher needs 2 forks to eat the spaghetti.
- The philosophers do not know each other's eating routine.

The problem: design a distributed protocol that prevents the philosophers from starvation, i e. enables each philosopher to eat infinitely often.

Not quite trivial.

Generalising the dining philosophers problem as a dynamic resource allocation problem



Generalising:

- Philosophers are **agents**
- Forks are **resources** (**resource units**)
- **Resource accessibility relation**
- Each agent has a need to fulfil (a goal to achieve): accumulate a prescribed number of resources.

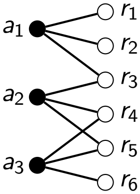
Generalized dining philosophers games

A **generalized dining philosophers (GDP) game** is a tuple

$$\mathcal{G} = (Agt, Res, d, Acc, Act, Rules) \text{ where:}$$

- Agt is a set of **agents**;
- Res is a set of **resource units**;
- $d : Agt \to \mathbb{N}^+$ is a **demand function**;
- $Acc \subseteq Agt \times Res$ is a **resource accessibility relation**.
- Act is a set of **possible actions**;
- $Rules$ is a set of **transition rules**;

Example



$d(a_i) = 2$
for $i \in \{1, 2, 3\}$

The intended goal for each agent a_i is **to acquire** $d(a_i)$ **resource units** (needed to carry out its task).

The actions and rules will be specified later.

Aim of this project

To develop a formal framework for specifying and verifying relevant individual and collective strategic abilities of agents in GDP games, such as "no deadlocks", or "no starvation", or e.g.:

"Agent a *can act strategically so as to ensure that she eventually reaches its goal (collects* $d(\mathsf{a})$ *resource units)."*

or (a collective goal):

"a_1 *and* a_2 *can act collaboratively so as to ensure that each of them reaches its goal (collects the needed resource units) infinitely often."*

or (a competitive goal):

"a_1 *and* a_2 *can act collaboratively so as to ensure that each of them reaches its goal (collects the needed resource units) infinitely often, whereas* a_3 *never reaches its goal."*

Generalised Dining Philosophers Games: technical introduction

Actions

Actions:

- req_r^a agent a requests resource r;
- rel_r^a agent a releases resource r;
- rel_{all}^a agent a releases all resources it holds;
- idle^a agent a does nothing.

An **action profile** is a mapping $ap : Agt \to Act$.

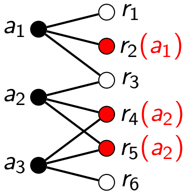
Configurations

A possible *state* of the game is called a **configuration**

$c : Res \to Agt^{+}$

Example

Given $\mathcal{G}$ as before the figure



graphically represents configuration where r_2 is held by a_1, r_4 is held by a_2 and r_5 by a_2.

Remark

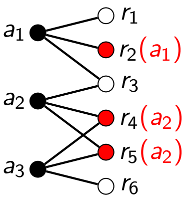
The number of configurations in a GDP game is, in general, exponential in the number of resources.

Transition rules and system dynamics

Given a configuration c and an action profile ap, (c, ap, c') is a **step** if:

1. ap **can be executed** in c, meaning:
   - agents can request only resources available in c;
   - if an agents a holds number $d(a)$ resources, it must perform rel^{a}_{all};
2. and the **resulting configuration** c' is such that:
   - the released resources become available in c';
   - if a resource is requested by one agent only, than that agent acquires it, otherwise no agent gets it.

Example

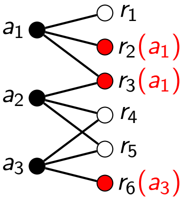


$$ap(a_1) = \mathrm{req}^{a_1}_{r_3}$$

$$ap(a_2) = \mathrm{rel}^{a_2}_{all}$$

$$ap(a_3) = \mathrm{req}^{a_3}_{r_6}$$

$$\Longrightarrow$$



Configuration graph

- **Transition function** of $\mathcal{G}$ is the set $\rho(\mathcal{G})$ of all game steps;
- $\mathfrak{G} = (\mathit{Conf}, \rho(\mathcal{G}))$ is the **configuration graph** of $\mathcal{G}$
- a **play** is an infinite sequence of configurations in $\mathfrak{G}$.

Competition and cooperation in GDP games

A GDP game is a *both competitive and cooperative* scenario, where agents may, but need not to, cooperate in pursuing their goal.

- On the one hand, each agent is interested in reaching their individual goal.
- However, that may become impossible if each agents acts selfishly (follows a greedy strategy), as that may lead to blocking resources.
- Thus, it is sometimes preferable for agents to cooperate by releasing resources before having reached their individual goals.
- Furthermore, some of them may wish to join forces and act in a coordinated way, as a coalition.

  That, inter alia, makes the analysis of GDP games quite non-trivial.
- Hence, the need for formal specification and algorithmic verification.

Remark: GDP games can also be regarded as "self-organising systems".

A logic for verifying GDP games

Our language $\mathcal{L}_{\mathrm{GDP}}$ is a slight variation of ATL:

$$\varphi ::= g_{a_i} \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \langle\langle \mathbf{A} \rangle\rangle \mathsf{X}\, \varphi \mid \langle\langle \mathbf{A} \rangle\rangle \mathsf{G}\, \varphi \mid \langle\langle \mathbf{A} \rangle\rangle \varphi_1 \mathsf{U}\, \varphi_2$$

where $\mathbf{A} \subseteq Agt$,
and g_{a_i} means that agent a_i currently holds at least $d(a_i)$ resource units (and has, therefore, reached its goal).

Strategies

For our language it suffices to consider *positional strategies*.

- a **(positional) strategy** for an agent a

$$\sigma_{\mathbf{a}} : Conf \to Act$$

which prescribes *executable* actions to the agent.

- a **joint (positional) strategy** for $A = \{a_1, \ldots, a_r\} \subseteq Agt$:

$$\sigma_{\mathbf{A}}(\sigma_{a_i}, \ldots, \sigma_{a_r})$$

is a tuple of individual strategies σ_{a_i}, for each $a_i \in A$.

Function $out(c, \sigma_A)$ returns the set of all plays in $Conf^{\omega}$ that can occur when agents in A follow the joint strategy σ_A from configuration c on.

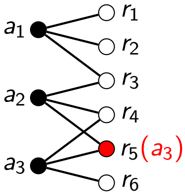
Formal semantics

$\mathcal{L}_{\mathrm{GDP}}$ is interpreted in GDP games as follows:

- $\mathfrak{G}, c \models g_{a_i}$ iff the number of resources a_i holds is $\geq d(a_i)$;
- $\wedge$, $\vee$ and $\neg$ are treated as usual;
- $\mathfrak{G}, c \models \langle\langle A \rangle\rangle \mathsf{X}\, \varphi$ iff there is a joint strategy σ_A, such that $\mathfrak{G}, \pi[1] \models \varphi$ for every path $\pi \in \mathit{out}(c, \sigma_A)$;
- $\mathfrak{G}, c \models \langle\langle A \rangle\rangle \mathsf{G}\, \varphi$ iff there is a joint strategy σ_A, such that $\mathfrak{G}, \pi[i] \models \varphi$ for every path $\pi \in \mathit{out}(c, \sigma_A)$ and for every $i \in \mathbb{N}$;
- $\mathfrak{G}, c \models \langle\langle A \rangle\rangle \varphi_1 \,\mathsf{U}\, \varphi_2$ iff there is a joint strategy σ_A, such that for every path $\pi \in \mathit{out}(c, \sigma_A)$:
  there exists $i \geq 0$ such that $\mathfrak{G}, \pi[i] \models \varphi_2$ and
  $\mathfrak{G}, \pi[j] \models \varphi_1$ for all j such that $0 \leq j < i$.

Example

$c_1 =$



$$\mathfrak{G}, c_1 \models \langle\langle a_1, a_3 \rangle\rangle \, \mathsf{G} \, (\langle\langle a_1 \rangle\rangle \, (\neg g_{a_2}) \, \mathsf{U} \, g_{a_1})$$

Model checking

ATL provides an algorithm for solving the global model checking problem:

Inputs:

- formula φ
- a GDP problem $\mathcal{G}$

Output:

- the **state extension** of φ in $\mathfrak{G}$

$$[\![\varphi]\!]_{\mathfrak{G}} = \{c \in \mathit{Conf} : \mathfrak{G}, c \models \varphi\}$$

Complexity

The ATL algorithm for global model checking problem applied to $\mathcal{L}_{\mathrm{GDP}}$ has worst-case time complexity exponential in the number of resources.

Since the number of resources can be large, this can be a problem.

Can we be more efficient?

Idea:

- Define a suitable **abstraction**: equivalence relation $\sim$ on configurations, that preserves truth of $\mathcal{L}_{\mathrm{GDP}}$ formulae;
- build the global model checking procedure to use that abstraction.

A natural abstraction

Observation:

- our logic cannot distinguish on atomic level configurations where agents hold the same number of resources

So, can we use

$$c_i \sim_{\#} c_j$$

iff

for each agent a,
the number of resources a holds in c_i
is the same it holds in c_j?

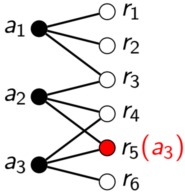
No! This is too coarse.

The abstraction $\sim_{\#}$ is too coarse

Example

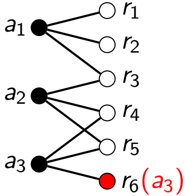
$$c_1 \sim_{\#} c_2$$

$c_1 =$



$\mathfrak{G}, c_1 \models \langle\!\langle a_3 \rangle\!\rangle \mathsf{X}\, g_{a_3}$ True

$c_2 =$



$\mathfrak{G}, c_2 \models \langle\!\langle a_3 \rangle\!\rangle \mathsf{X}\, g_{a_3}$ False

A correct abstraction

A finer abstraction is required.

1. We first define an equivalence relation on resources

$$\mathbf{r_i \approx r_J}$$

iff
r_i and r_j are accessible by the same subset of agents

2. We then define

$$\mathbf{c_1 \sim c_2}$$

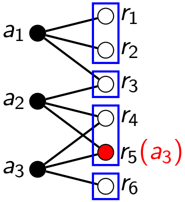
iff
for each agent a and
for each equivalence class of resource $R \in Res/\approx$
the number of resources from R
that a holds in c_1 is the same as in c_2

A sound and complete abstraction

Example

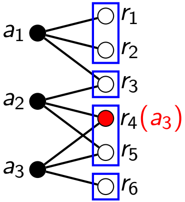
$c_1 \approx c_2$

$c_1 =$



$\mathfrak{G}, c_1 \models \langle\langle a_3 \rangle\rangle \, \mathsf{X} \, g_{a_3}$ True

$c_3 =$



$\mathfrak{G}, c_3 \models \langle\langle a_3 \rangle\rangle \, \mathsf{X} \, g_{a_3}$ True

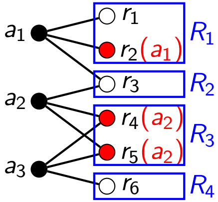
Interval expressions

We symbolically represent sets of configurations with expressions:

$$\alpha ::= \bigwedge_{a \in Agt} \bigwedge_{R \in \mathcal{R}} (a, R)[l_R^a, l_R^a] \mid \alpha_1 \vee \alpha_2$$

and $\|\alpha\|_{\mathfrak{G}}$ denotes the set of configurations "contained" in α

Example



is contained in:

$$(a_1, R_1)[1,1] \wedge (a_1, R_2)[0,0] \wedge$$
$$(a_2, R_2)[0,0] \wedge (a_2, R_3)[2,2] \wedge$$
$$(a_3, R_3)[0,0] \wedge (a_3, R_4)[0,0]$$

A symbolic model checking algorithm for $\mathcal{L}_{\mathrm{GDP}}$

We develop a *symbolic* global model checking algorithm for $\mathcal{L}_{\mathrm{GDP}}$.

Given

- a game $\mathcal{G}$
- a formula φ

it returns

- the **interval constraint expression** $\alpha(\mathcal{G}, \varphi)$

Theorem

For each game $\mathcal{G}$ and formula $\varphi \in \mathcal{L}_{\mathrm{GDP}}$ we have:

$$c \in [\![\varphi]\!]_{\mathfrak{G}} \text{ iff } c \in \alpha(\mathcal{G}, \varphi)$$

Complexity

The symbolic global model checking algorithm runs in time at most *double exponential in the number of agents* but *polynomial in the number of resources*.

Lecture 4.1: Closing remarks and the read ahead

This project is still in an early state of development. Much yet to be done.

On the technical side:

- To obtain more refined complexity results.
  (The double exponential case seems to never actually happen.)
- Can we do better? Is our model-checking algorithm optimal?
- Find analytic solutions for important special cases.

On the conceptual side:

- Explore the cases with agents' incomplete and imperfect information.
- Gam-theoretic analysis: identify and analyse the equilibria, design socially optimal equilibria, etc.
- Extend the framework to one where resources are autonomous agents themselves. *Clients/Bankers problem*.

END OF LECTURE 4.1