Machines Climbing Pearl's Ladder of Causation

Lecture III - Causal Effect Identification & Estimation

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Outline

- Structure Causal Model (SCM)
- Causal Bayesian Network (CBN) / Causal Diagrams
- Effect Identification given a Causal Diagram
  - Identification in Markovian Models
  - Identification in Semi-Markovian Models
    - Adjustment Formula: Parent, Backdoor Criterion
    - Front-Door Criterion
    - General Tools: Do-Calculus & ID-Algorithm
- Effect Identification in the Markov Equivalence Class
- Current Challenges and Open Problems

Prediction vs Effect of Interventions

Statistical Association vs Causation

Predictive Tasks

Task: Can I **guess** the size of a fire by **observing** the number of firefighters?

Yes!

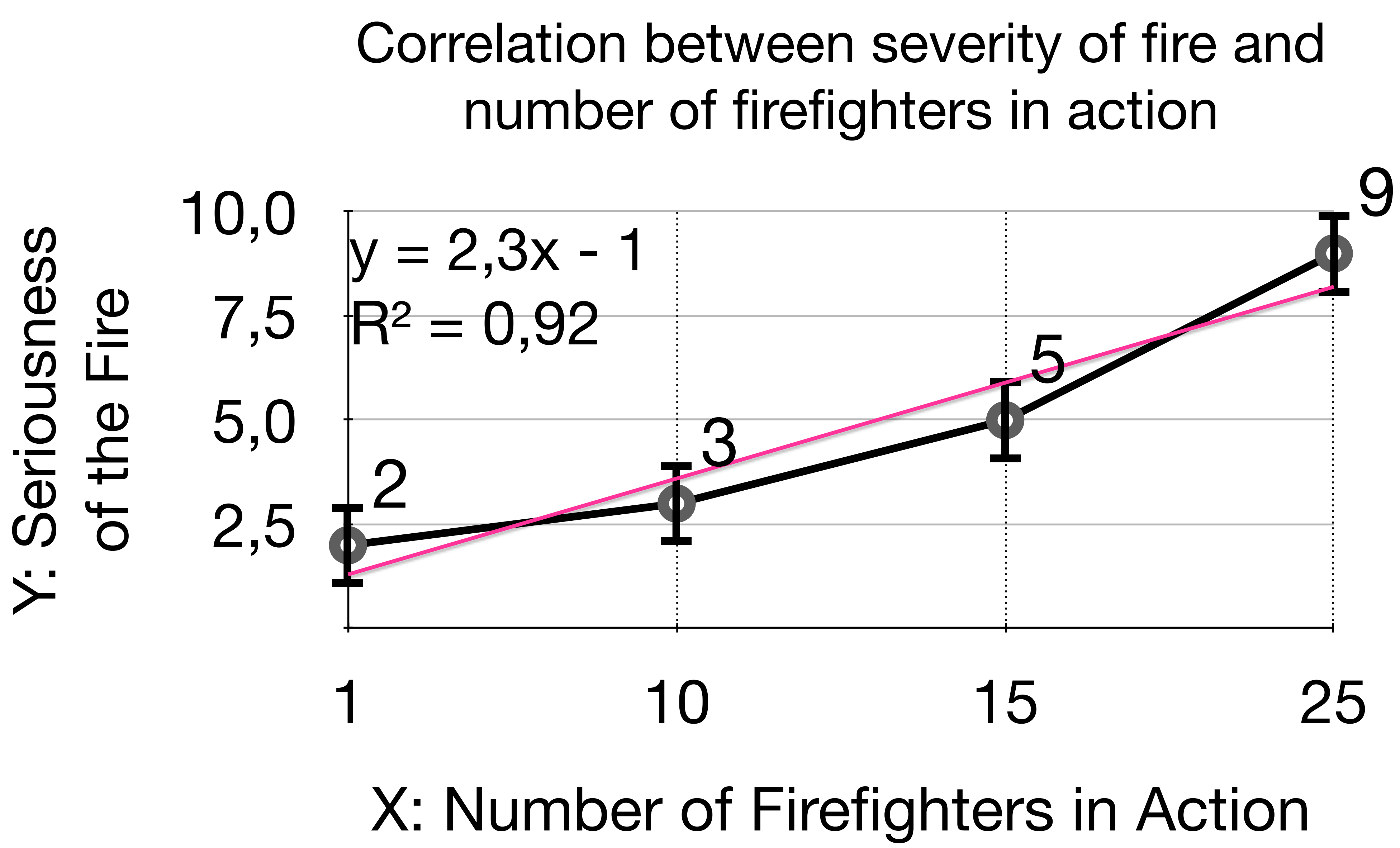
X: **Number of firefighters in action**

Y: **Size of the (initial) fire**

$\rho_{XY} \neq 0 \implies X$ **is a good predictor of** Y

$$P(Y = y \mid X = x) \neq P(Y = y)$$

Observational Probability Distribution



Positive Correlation:

More firefighters mean a bigger fire;
Fewer firefighters mean a smaller fire.

Prediction ⇒ Decision-Making?

Should we reduce the number of firefighters to decrease the size of the fire?

Misleading correlation: It is the size of the fire that determines the number of firefighters needed, not the other way around.

Causal Effect ≡ Effect of an Intervention

The causal direction is determined by understanding the underlying reality.

X: **Number of firefighters in action**

Y: **(Initial) Severity of the fire**

$$\begin{cases} X = f_X(Y, U_X) \\ Y = f_Y(U_Y) \end{cases}$$

Underlying Structural Causal Model (SCM)

Y is not a function of X

In other words, X **is not a cause of** Y

Changing the number of firefighters through an action/intervention on X, $do(X = x)$, does not affect the initial size of the fire (Y).

Structural Causal Model (SCM)

EXPLAINABILITY AND THE DATA GENERATING MODEL

Structural Causal Model (SCM)

Definition: A structural causal model $\mathcal{M}$ (or, data generating model) is a tuple $\langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$, where

- $\mathbf{V} = \{V_1, \ldots, V_n\}$: are endogenous variables
- $\mathbf{U} = \{U_1, \ldots, U_m\}$: are exogenous variables
- $\mathcal{F} = \{f_1, \ldots, f_n\}$: are functions determining $\mathbf{V}$, i.e., $v_i \leftarrow f_i(pa_i, u_i)$, where
  - $Pa_i \subseteq \mathbf{V}$ are endogenous causes (parents) of V_i
  - $U_i \subseteq \mathbf{U}$ are exogenous causes of V_i.
- $P(\mathbf{U})$ is the probability distribution over $\mathbf{U}$.

Assumption: $\mathcal{M}$ is recursive, i.e., there are no feedback (cyclic) mechanisms.

Structural Equation Model (SEM)

$$\mathscr{M} = \begin{cases} \mathbf{V} = \{X, Y, Z\} \\ \mathbf{U} = \{\epsilon_X, \epsilon_Y, \epsilon_Z\} \\ \mathscr{F} = \begin{cases} Z = \beta_{Z0} + \epsilon_Z \\ X = \beta_{X0} + \beta_{XZ} Z + \epsilon_X \\ Y = \beta_{Y0} + \beta_{YZ} Z + \beta_{YX} X + \epsilon_Y) \end{cases} \\ \mathbf{U} \sim \mathscr{N}\left(\mathbf{0}, \Sigma = \begin{bmatrix} \sigma_X & 0 & 0 \\ 0 & \sigma_Y & 0 \\ 0 & 0 & \sigma_Z \end{bmatrix}\right) \end{cases}$$

- **Pre-specified causal order**
- **Linear functions**
- **Normal distribution**
- **Markovianity / Causal Sufficiency**: Error terms in $\mathbf{U}$ are independent of each other (diagonal covariance matrix).

Full specification of an SCM requires parametric and distributional assumptions.

Estimation of such models usually requires strong assumptions (e.g., Markovianity).

Statistical Association vs Causation

Pre-Interventional/ Observational SCM

$$\mathcal{M} = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = f_X(U_X, U_{XY}) \\ Y = f_Y(X, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Observational Distribution

$$P(\mathbf{V}) \doteq P_{\mathcal{M}}(\mathbf{V})$$

Can we ***predict*** better the value of Y after ***observing*** that $X = x$?

$P(Y = y \mid X = x) \neq P(Y = y) \implies X$ is ***correlated*** to Y

$do(X = x)$

$\neq$

Post-Interventional / Interventional SCM

$$\mathcal{M}_x = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = x \\ Y = f_Y(x, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Interventional Distribution

$$P(\mathbf{V} \mid do(X = x)) \doteq P_{\mathcal{M}_x}(\mathbf{V})$$

Can we ***predict*** better the value of Y after ***making an intervention*** $do(X = x)$?

$\exists x \text{ s.t. } P_{\mathcal{M}_x}(Y = y) \neq P(Y = y) \implies X$ is ***a cause*** of Y

Causal Bayesian Network

A DAG, possibly with latent confounders (ADMG),
representing the **causal and confounding relationships**
implied by an SCM

CBN: Encoder of Structural Causal Knowledge

Structural Causal Model (SCM)

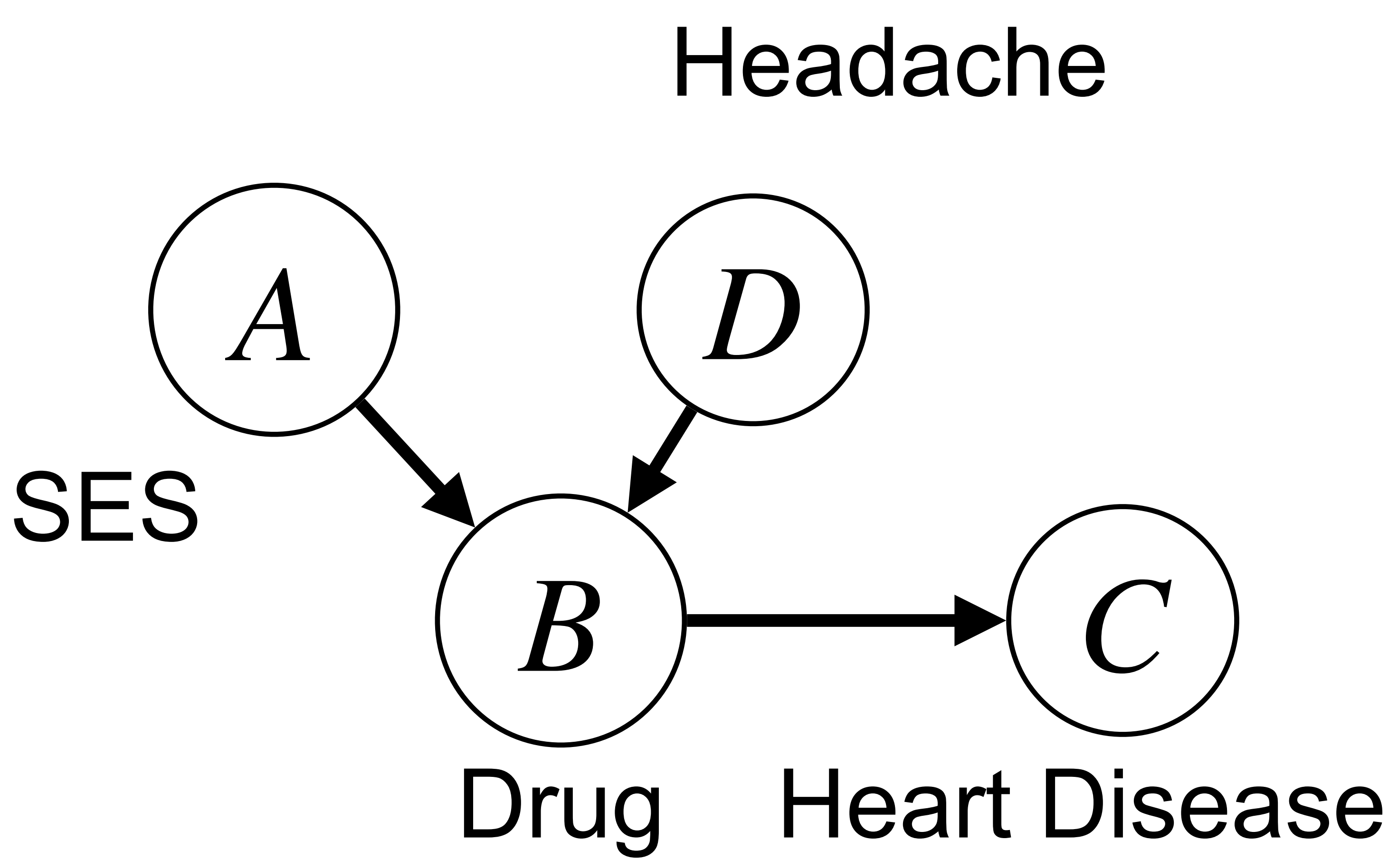
$\mathcal{M} = \langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$

$$\mathcal{M} = \begin{cases} \mathbf{V} = \{A, B, C, D\} \\ \mathbf{U} = \{U_A, U_B, U_C, U_D, U_{CD}\} \\ \mathcal{F} = \begin{cases} A \leftarrow f_A(U_A) \\ B \leftarrow f_B(A, D, U_B) \\ D \leftarrow f_Z(U_D, U_{CD}) \\ C \leftarrow f_X(B, U_C, U_{CD}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

→

Induced Causal Bayesian Network (CBN)

Causal Diagram



An SCM $\mathcal{M} = \langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$ induces a causal diagram such that, **for every $V_i, V_j \in \mathbf{V}$**:

$V_i \rightarrow V_j$, if V_i appears as argument of $f_j \in \mathcal{F}$.

CBN: Encoder of Structural Causal Knowledge

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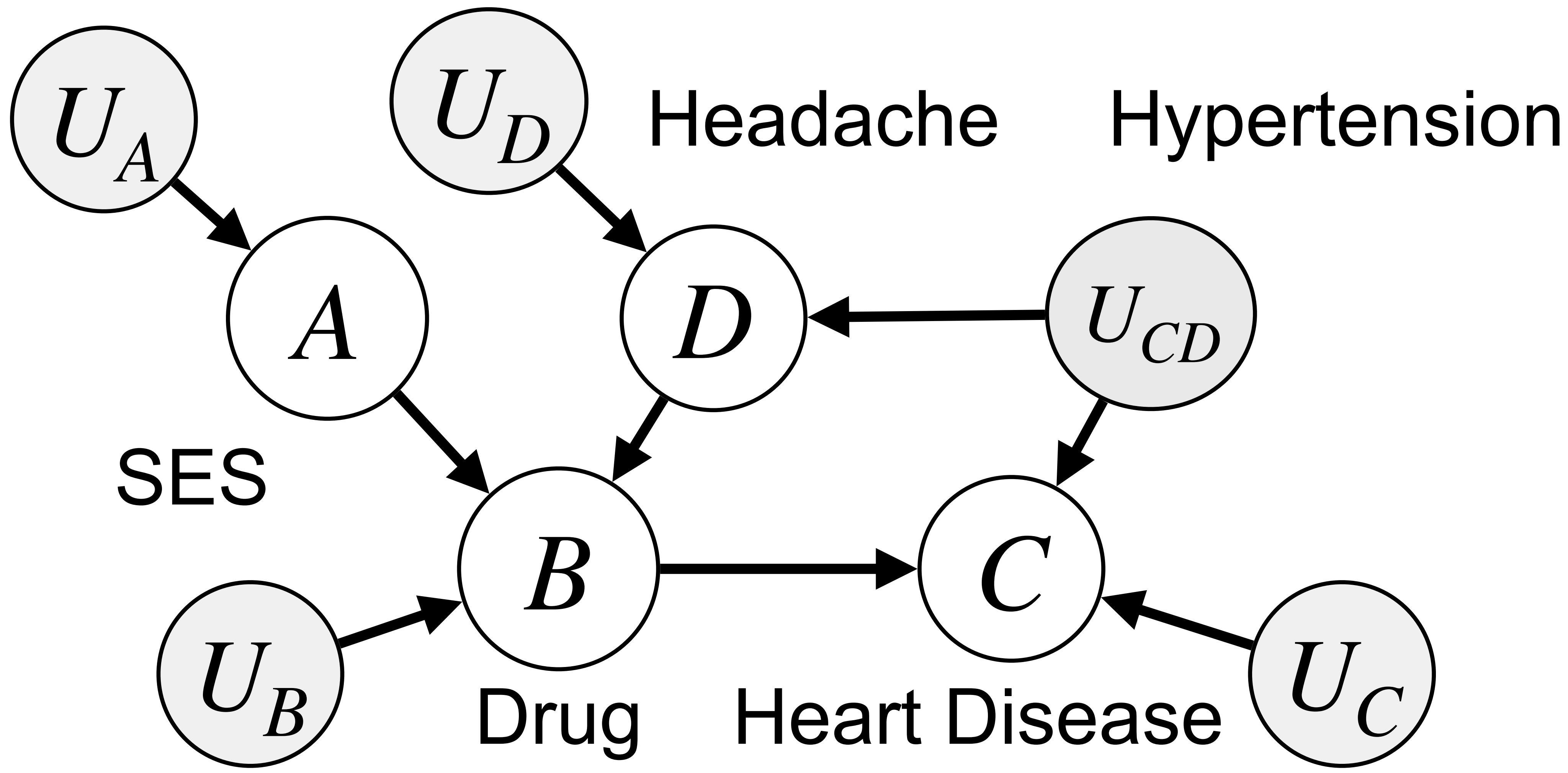
Induced Causal Bayesian Network (CBN)

Causal Diagram



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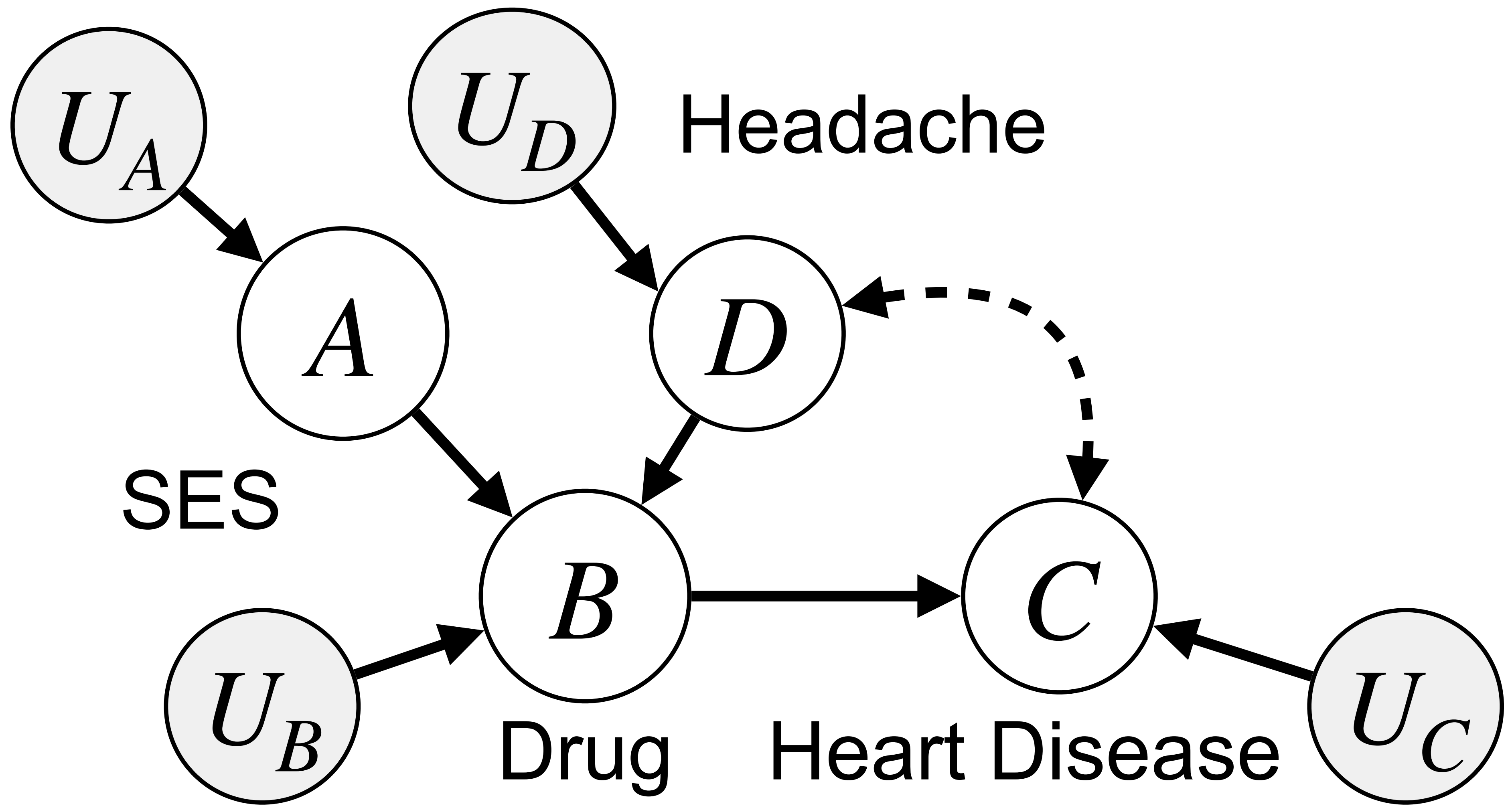
Induced Causal Bayesian Network (CBN)

Causal Diagram



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$V_i \rightarrow V_j$, if V_i appears as argument of $f_j \in \mathcal{F}$.

$V_i \leftarrow\text{-}\text{-}\rightarrow V_j$ if the corresponding $U_i, U_j \in \mathbf{U}$ are correlated or f_i , f_j share some argument $U \in \mathbf{U}$.

CBN: Encoder of Structural Causal Knowledge

Structural Causal Model (SCM)

$$\mathcal{M} = \langle \mathbf{V}, \mathbf{U}, \mathcal{F}, P(\mathbf{u}) \rangle$$

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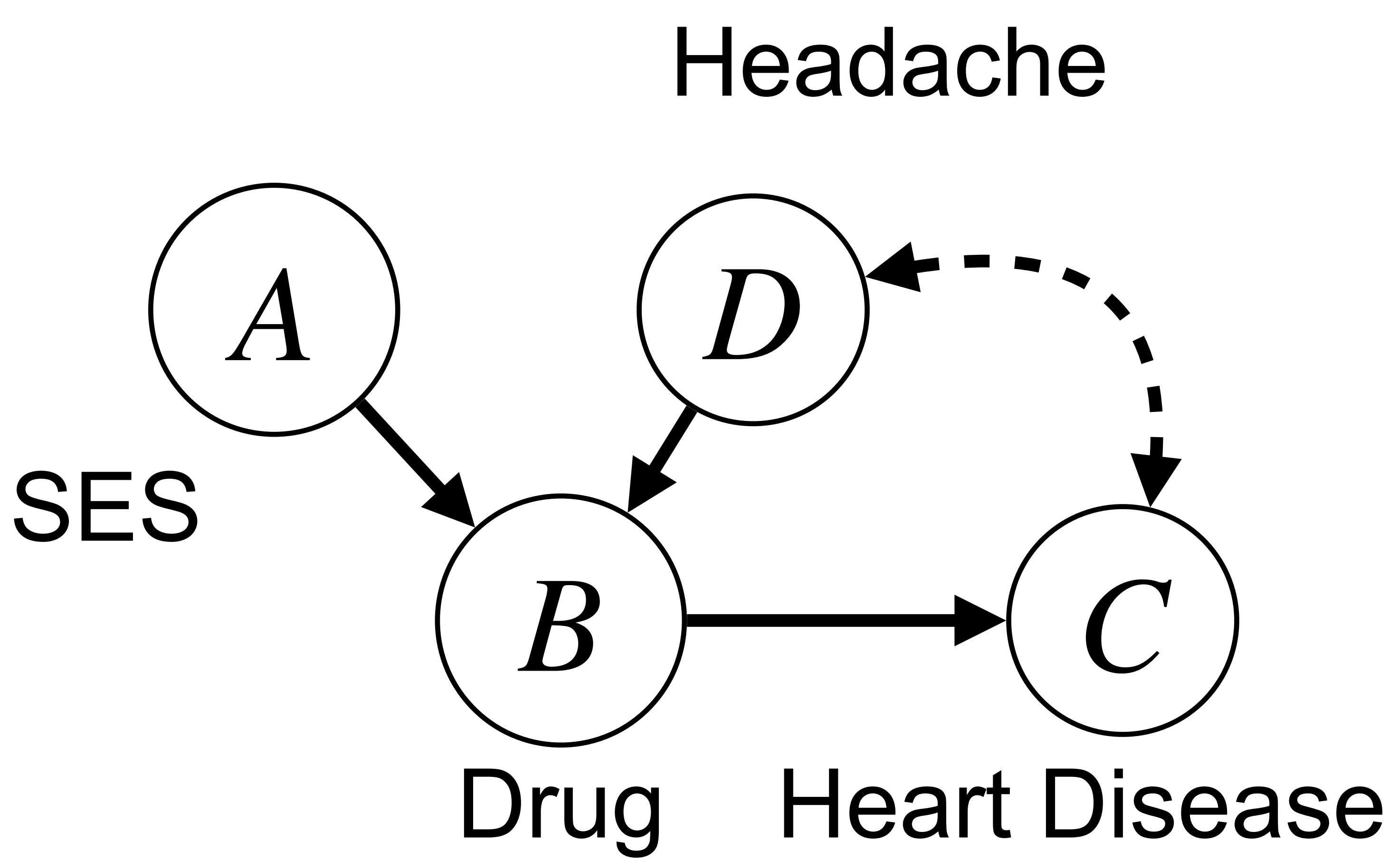
Induced Causal Bayesian Network (CBN)

Causal Diagram



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$V_i \rightarrow V_j$, if V_i appears as argument of $f_j \in \mathcal{F}$.

$V_i \leftarrow\!\!\dashrightarrow V_j$ if the corresponding $U_i, U_j \in \mathbf{U}$ are correlated or f_i , f_j share some argument $U \in \mathbf{U}$.

CBN: Encoder of Structural Causal Knowledge

Let $\mathbf{P}_*$ be the collection of all interventional distributions $P(\mathbf{V} \mid do(\mathbf{x}))$, $\mathbf{X} \subseteq \mathbf{V}$, including the null (observational) distribution $P(\mathbf{V})$.

An Acyclic Directed Mixed Graph (ADMG) G is a CBN for $\mathbf{P}_*$ if for every intervention $do(\mathbf{X} = \mathbf{x})$, $\mathbf{X} \subseteq \mathbf{V}$, if it hold:

Interventional Distribution

$$P(\mathbf{V} \mid do(X = x)) \doteq P_{\mathscr{M}_x}(\mathbf{V})$$

$$= \sum_{\mathbf{u}} \prod_{V_i \in \mathbf{V} \setminus \mathbf{X}} P(v_i \mid pa_i, u_i) P(\mathbf{u}) \Big|_{\mathbf{X}=\mathbf{x}}$$

Truncated factorization implied by the SCM $\mathscr{M}_x$.

Semi-Markov relative to $G_{\overline{\mathbf{X}}}$

Statistical Association vs Causation

Pre-Interventional/ Observational SCM

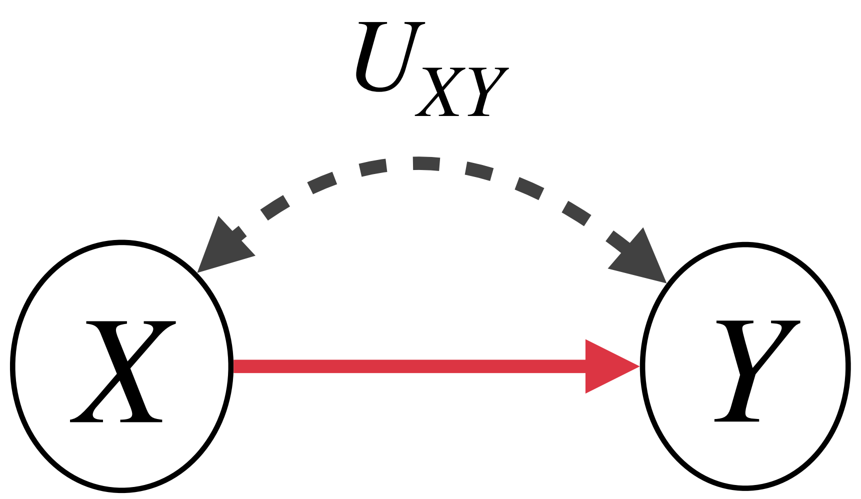
$$\mathcal{M} = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = f_X(U_X, U_{XY}) \\ Y = f_Y(X, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Loss of Information

Observational Data

$P_{\mathcal{M}}(\mathbf{V})$

Observational Distribution



Observational Causal Diagram

$do(X = x)$

Post-Interventional / Interventional SCM

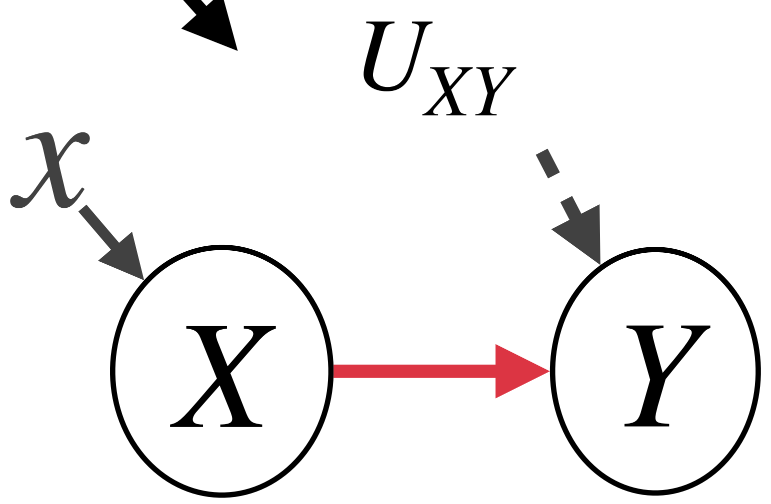
$$\mathcal{M}_x = \begin{cases} \mathbf{V} = \{X, Y\} \\ \mathbf{U} = \{U_{XY}, U_X, U_Y\} \\ \mathcal{F} = \begin{cases} X = x \\ Y = f_Y(x, U_Y, U_{XY}) \end{cases} \\ P(\mathbf{U}) \end{cases}$$

Loss of Information

Interventional Data

$P_{\mathcal{M}_x}(\mathbf{V}) \doteq P(\mathbf{V} \mid do(x))$

Interventional Distribution

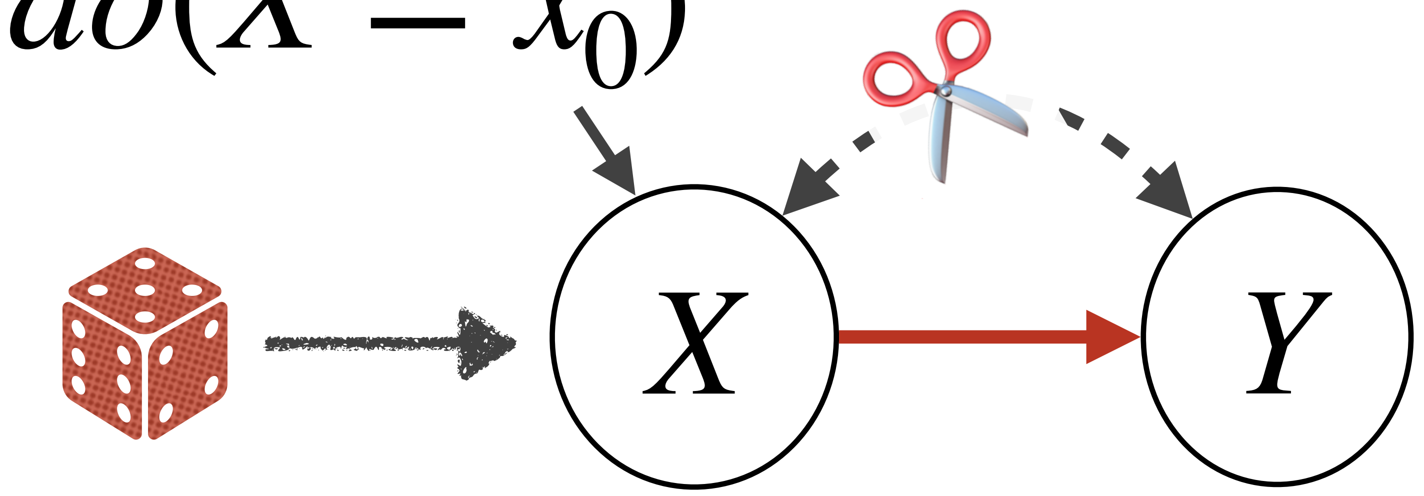


Interventional Causal Diagram

Randomized Experiments

A well accepted way to access $P(Y \mid do(X = x))$ is through a *perfectly realized* Randomized Experiments / Control Trials (e.g. RCT):

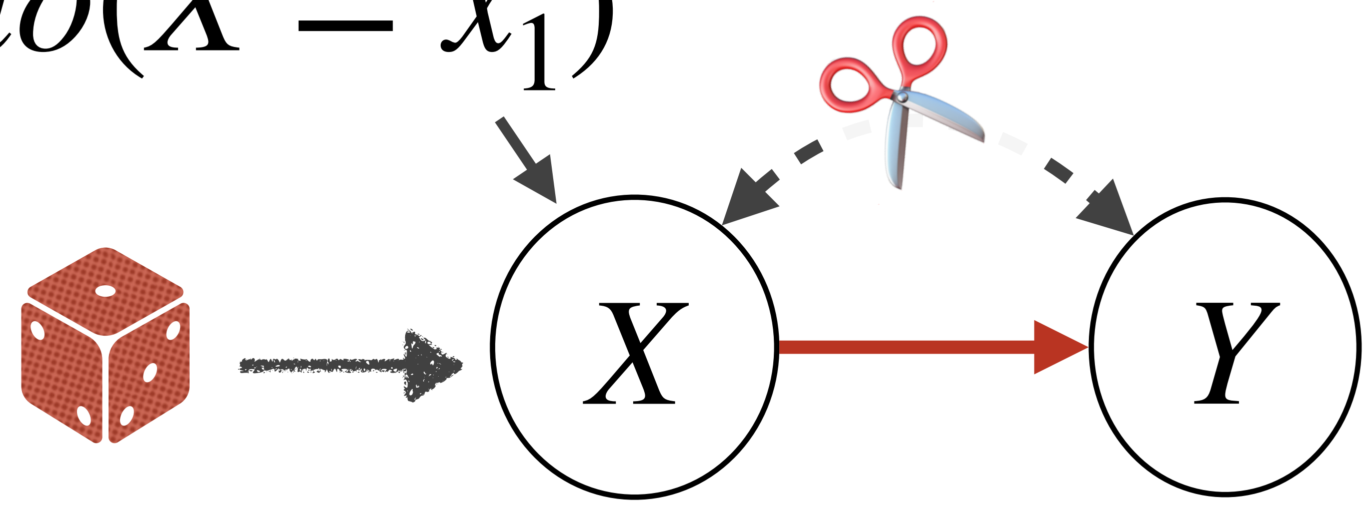
$do(X = x_0)$



$\mathbb{E}[Y \mid do(X = x_0)]$

Randomization of the X's assignment

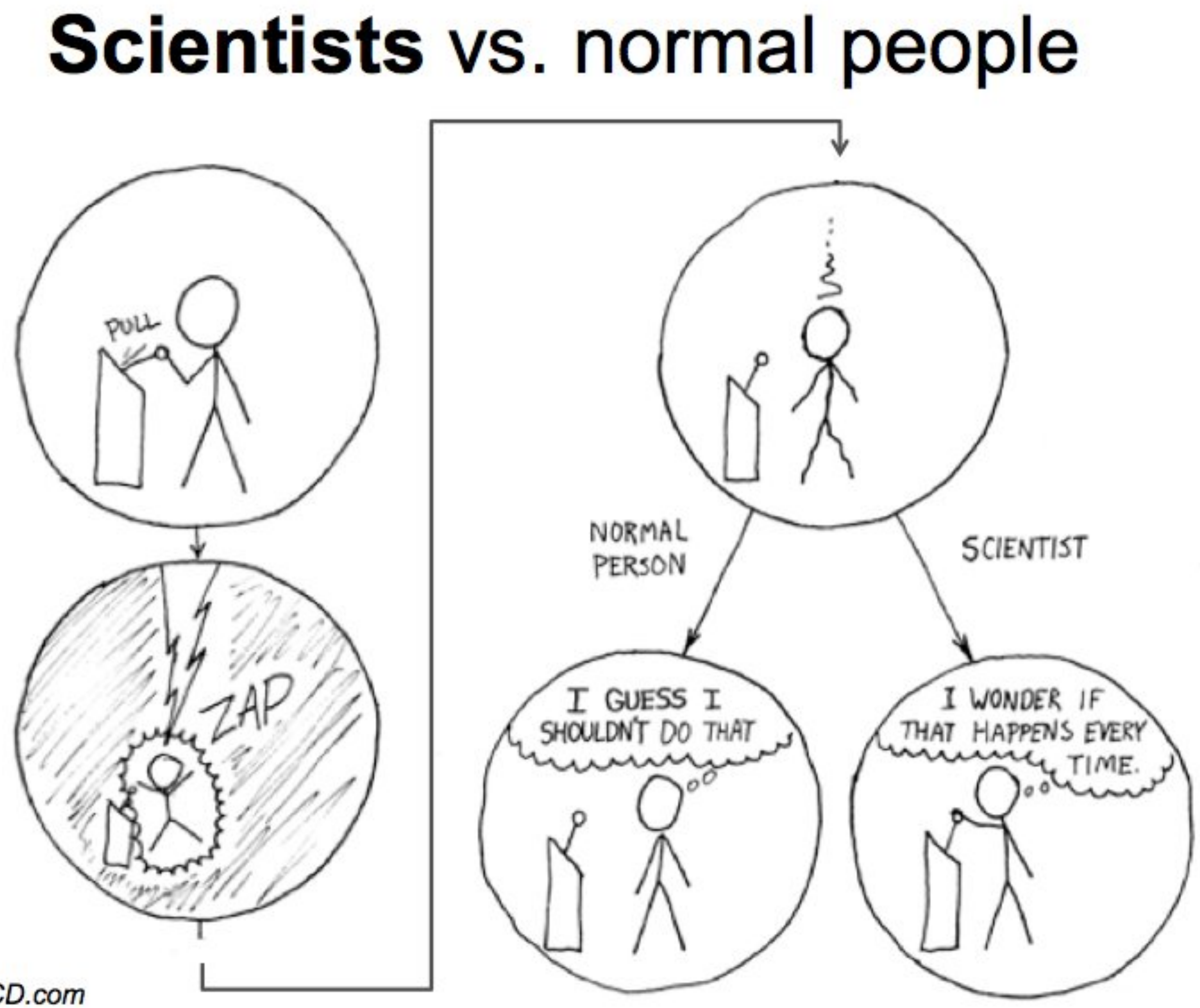
$do(X = x_1)$



$\mathbb{E}[Y \mid do(X = x_1)]$

Average Causal Effect: $\mathbb{E}[Y \mid do(X = x_0)] - \mathbb{E}[Y \mid do(X = x_1)]$

Can we always conduct randomized experiments?



From XKCD.com

- Ethical concerns
- Practical limitations
- Logistical challenges

Causal Effect Identification given a Causal Diagram / CBN

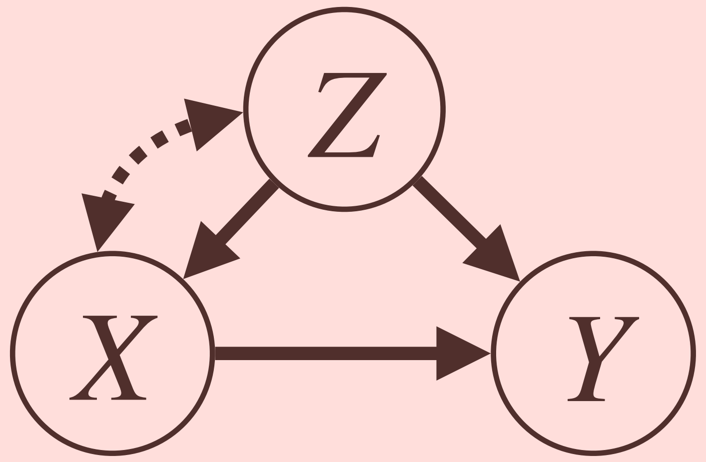
Classical Causality Pipeline from a Causal Diagram

Structural knowledge available

1 **Query**

$P(y \mid do(x))$

2 **Causal Contraints**



3 $P(x, m, y)$

Observational Distribution

Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

$$\mathbb{E}(y \mid do(x)) = \sum_z \mathbb{E}(y \mid x, z) P(z)$$

Interventional Distribution ← Available Distributions

👍

Observational Data

$$\rightarrow \hat{\mathbb{E}}(y \mid do(x)) = \sum_z \hat{\mathbb{E}}((y \mid x, z) \hat{P}(z)$$

Causal Modeling / Causal Discovery → Causal Effect Identification → Causal Effect Estimation

Causal Effect

The **causal effect** of a (set of) treatment variable(s) $\mathbf{X}$ on a (set of) outcome variable(s) $\mathbf{Y}$ is a quantity derived from $P(\mathbf{Y} \mid do(\mathbf{X}))$ that tells us how much $\mathbf{Y}$ changes due to an intervention $do(\mathbf{X} = \mathbf{x})$.

Examples:

- *Average Treatment Effect* (ATE) for discrete treatments:

$$\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x}')] - \mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})],$$

where $\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})] = \sum_{\mathbf{y} \in \Omega_{\mathbf{Y}}} \mathbf{y} P(\mathbf{y} \mid do(\mathbf{x}))$

defined for two treatment levels $\mathbf{x}'$ and $\mathbf{x}$ of $\mathbf{X}$.

- *Average Treatment Effect* (ATE) for continuous treatments,

$$\frac{\partial \mathbb{E}[Y_i \mid do(X_j = x_j)]}{\partial x_j}, \text{ for all } Y_i \in \mathbf{Y}, \text{and } X_j \in \mathbf{X}.$$

Jacobian of $\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})]$, where

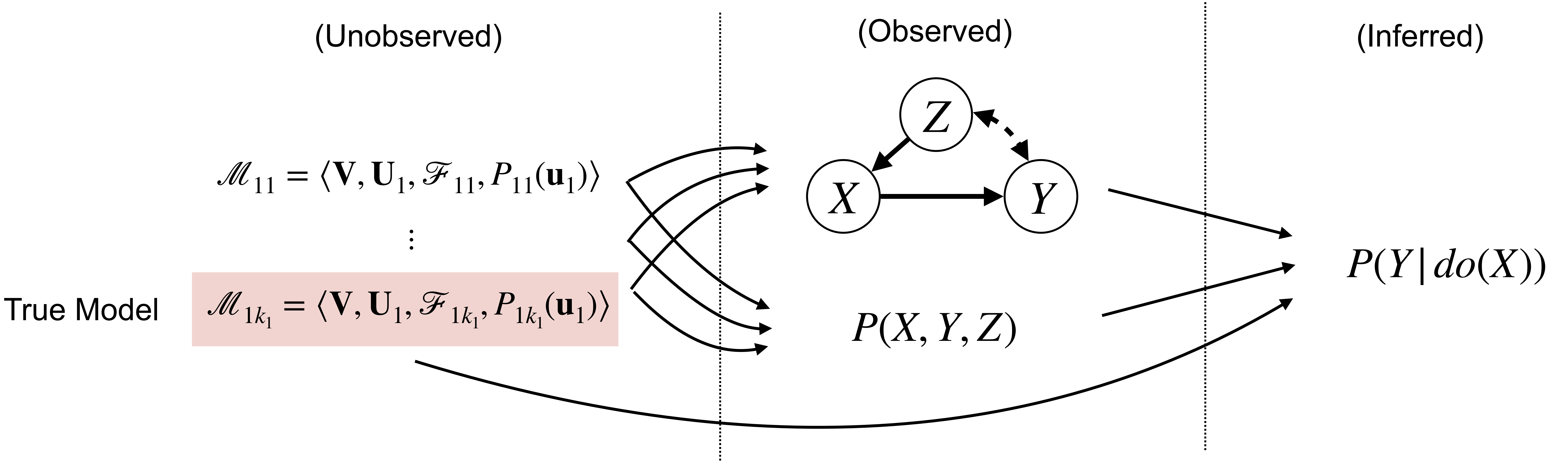
$$\mathbb{E}[\mathbf{Y} \mid do(\mathbf{X} = \mathbf{x})] = \int_{\Omega_{\mathbf{y}}} \mathbf{y} P(\mathbf{y} \mid do(\mathbf{x})) d\mathbf{y},$$

and $\Omega_{\mathbf{Y}}$ is the space of all possible values that $\mathbf{Y}$ might take on

The derivative shows the rate of change of $\mathbf{Y}$ w.r.t. $do(\mathbf{X} = \mathbf{x})$

The Effect Identification Problem

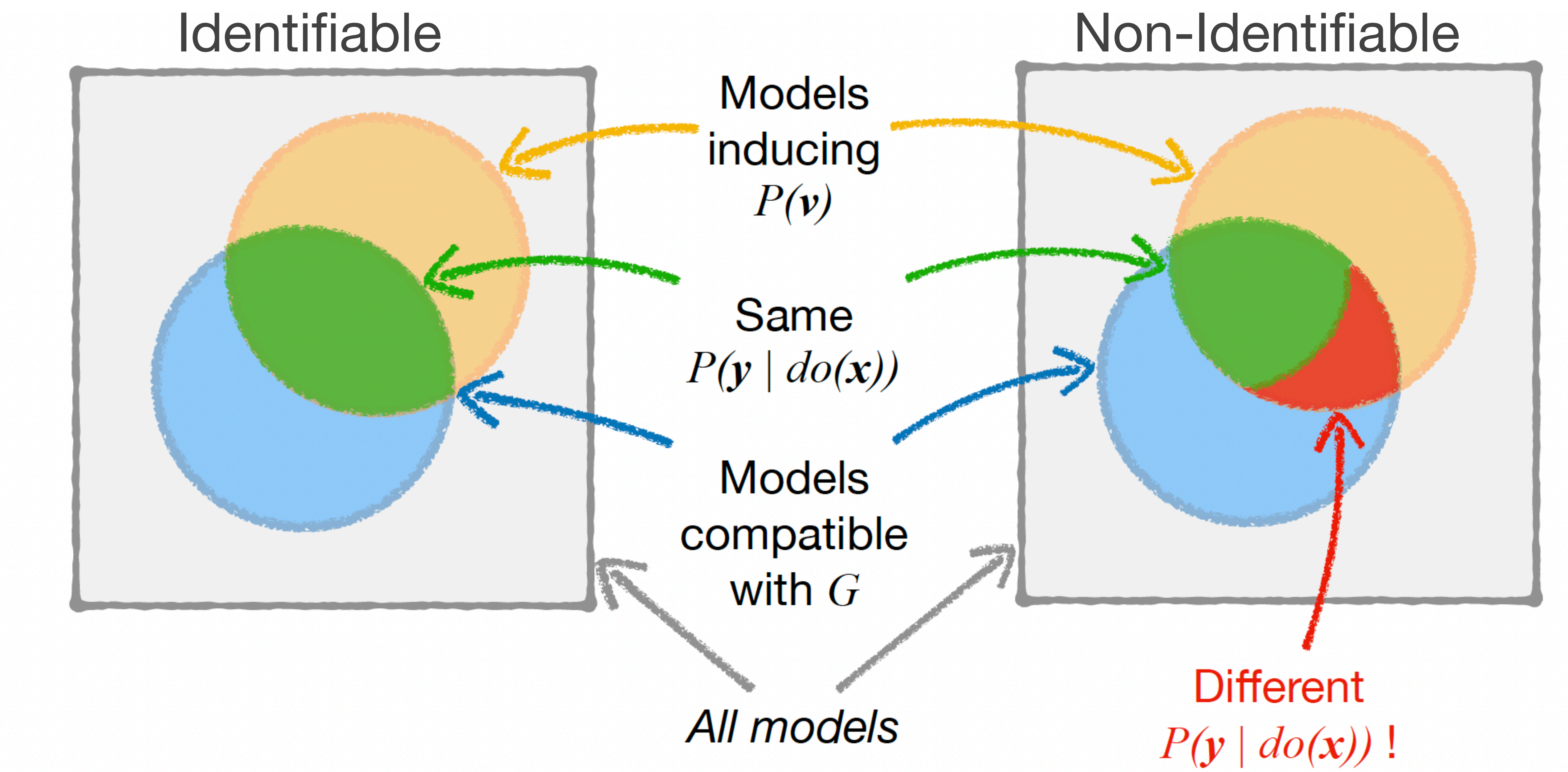
Causal Effect Identifiability: The causal effect of a (set of) treatment variable(s) $\mathbf{X}$ on a (set of) outcome variable(s) $\mathbf{Y}$ is said to be identifiable from a causal diagram G and the probability of the observed variables $P(\mathbf{V})$ if the interventional distribution $P(\mathbf{Y} \mid do(\mathbf{X}))$ is *uniquely computable,* i.e., if for every pair of SCMs $\mathcal{M}_1$ and $\mathcal{M}_2$ that induce G and $P^{\mathcal{M}_1}(\mathbf{V}) = P^{\mathcal{M}_2}(\mathbf{V}) = P(\mathbf{V}) > 0$, $P^{\mathcal{M}_1}(\mathbf{Y} \mid do(\mathbf{X})) = P^{\mathcal{M}_2}(\mathbf{Y} \mid do(\mathbf{X})) = P(\mathbf{Y} \mid do(\mathbf{X}))$.



In words, causal effect identifiability means that, no matter the form of true SCM, for all models $\mathcal{M}$ agreeing with $\langle G, P(\mathbf{V}) \rangle$, they also agree in $P(\mathbf{y} \mid do(\mathbf{x}))$.

The Effect Identification Problem

Causal Effect Identifiability: The causal effect of a (set of) treatment variable(s) $\mathbf{X}$ on a (set of) outcome variable(s) $\mathbf{Y}$ is said to be identifiable from a causal diagram G and the probability of the observed variables $P(\mathbf{V})$ if the interventional distribution $P(\mathbf{Y} \mid do(\mathbf{X}))$ is *uniquely computable,* i.e., if for every pair of SCMs $\mathcal{M}_1$ and $\mathcal{M}_2$ that induce G and $P^{\mathcal{M}_1}(\mathbf{V}) = P^{\mathcal{M}_2}(\mathbf{V}) = P(\mathbf{V}) > 0$, $P^{\mathcal{M}_1}(\mathbf{Y} \mid do(\mathbf{X})) = P^{\mathcal{M}_2}(\mathbf{Y} \mid do(\mathbf{X})) = P(\mathbf{Y} \mid do(\mathbf{X}))$.



In words, causal effect identifiability means that, no matter the form of true SCM, for all models $\mathcal{M}$ agreeing with $\langle G, P(\mathbf{V}) \rangle$, they also agree in $P(\mathbf{y} \mid do(\mathbf{x}))$.

Tools for Causal Identification

1. Truncated Factorization / G-computation formula

Markovian Models

2. Graphical criteria
    1. Parent adjustment
    2. Backdoor Adjustment
    3. Front-door Adjustment

A few interesting (albeit still constrained) scenarios

3. Do-Calculus (a.k.a Causal Calculus)
4. Identify Algorithm (a.k.a. ID algorithm)

General Semi-Markovian Scenarios

Pearl, J. (2000). Causality: Models, Reasoning, and Inference. Cambridge University Press, New York. http://dx.doi.org/10.1017/CBO9780511803161

Jin Tian. Studies in causal reasoning and learning. PhD thesis, University of California, Los Angeles, 2002.

Identification in Markovian Models

Truncated Factorization — Markovian: Let G be a causal diagram for the collection $\mathbf{P}_*$ of all interventional distributions $P_{\mathbf{x}}(\mathbf{V})$, for any $\mathbf{X} \subseteq \mathbf{V}$. It follows that $P_{\mathbf{x}}(\mathbf{V})$ factorizes as:

$$P_{\mathbf{x}}(\mathbf{v}) \doteq P(\mathbf{v} \mid do(\mathbf{x})) = \prod_{V_i \in \mathbf{V} \setminus \mathbf{X}} P_{\mathbf{x}}(v_i \mid pa_i) \Big|_{\mathbf{X}=\mathbf{x}}$$

Follows from $P_{\mathbf{x}}(\mathbf{v}) \doteq P(\mathbf{v} \mid do(\mathbf{x}))$ being *Markov* relative to $G_{\overline{\mathbf{X}}}$

$$= \prod_{V_i \in \mathbf{V} \setminus \mathbf{X}} P(v_i \mid pa_i) \Big|_{\mathbf{X}=\mathbf{x}}$$

Markovian SCMs have the modularity property, i.e., $P_{\mathbf{x}}(v_i \mid pa_i) = P(v_i \mid pa_i)$

Causal Effect of $\mathbf{X}$ on $\mathbf{Y}$:

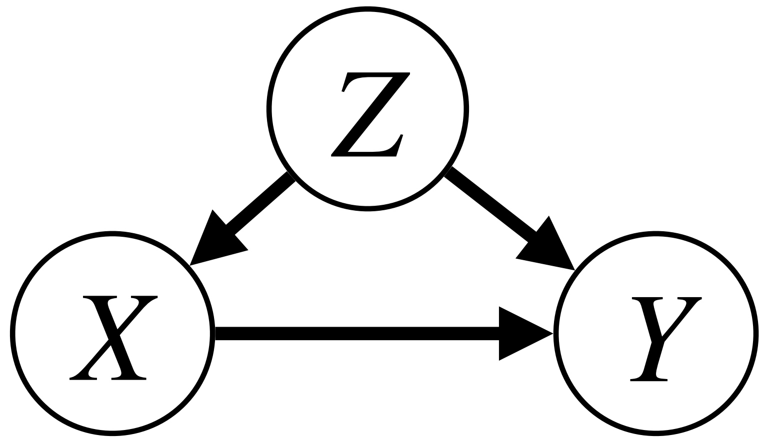
$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{V} \setminus (\mathbf{Y} \cup \mathbf{X})} \prod_{V_i \in \mathbf{V} \setminus \mathbf{X}} P(v_i \mid pa_i) \Big|_{\mathbf{X}=\mathbf{x}}$$

- In Markovian Models, the joint interventional distribution (and hence any causal effect) is always identifiable.
- This factorization is a.k.a "manipulation theorem" (Spirtes et al. 1993) or G-computation (Robins 1986, p. 1423).

Example: Identifiable Effect

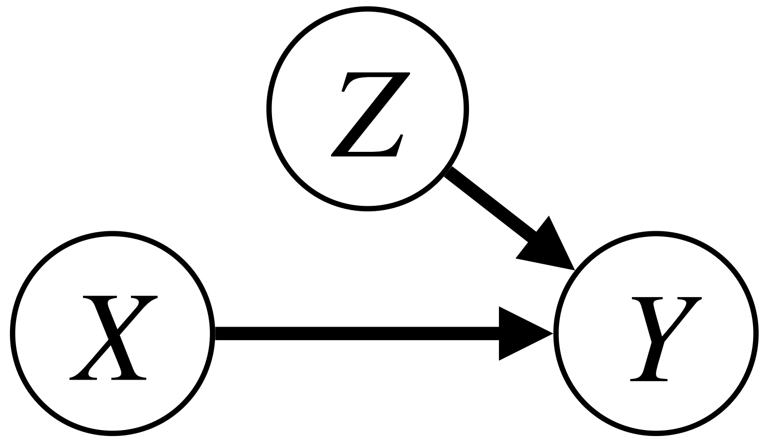
Causal Effect of $\mathbf{X}$ on $\mathbf{Y}$: $$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{V} \setminus (\mathbf{Y} \cup \mathbf{X})} \prod_{V_i \in \mathbf{V} \setminus \mathbf{X}} P_{\mathbf{x}}(v_i \mid pa_i) \Big|_{\mathbf{X}=\mathbf{x}}$$

G



$do(X = x)$

$G_{\overline{X}}$



$$P(x, y, z) = P(z)P(x \mid z)P(y \mid x, z)$$

$$P(y, z \mid do(x)) = P(z)P(y \mid x, z)$$

$$\implies P(y \mid do(x)) = \sum_{z} P(z)P(y \mid x, z)$$

Identification in Semi-Markovian Models

Adjustment over parents:

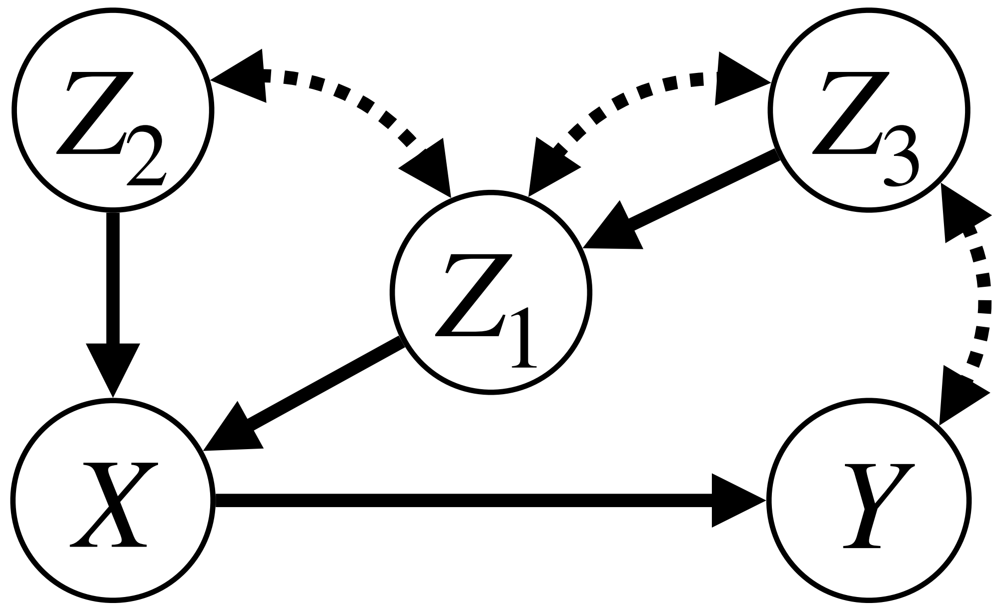
Let G be a causal graph with **no unmeasured parents**.

Then, the effect of $\mathbf{X}$ on $\mathbf{Y}$ is given by:

$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{pa_x}} P\left(\mathbf{y} \mid \mathbf{x}, \mathbf{pa_x}\right) P\left(\mathbf{pa_x}\right)$$

Proof follows from the truncated factorization for Markovian models!

$\mathbf{X} = \{X\}$
$\mathbf{Y} = \{Y\}$
$\mathbf{Pa_X} = \{Z_1, Z_2\}$



$Pa_x = \{Z_1, Z_2\}$

$$P(y \mid do(x)) = \sum_{z_1, z_2} P\left(y \mid x, z_1, z_2\right) P\left(z_1, z_2\right)$$

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Let G be a causal graph with **no unmeasured parents**.

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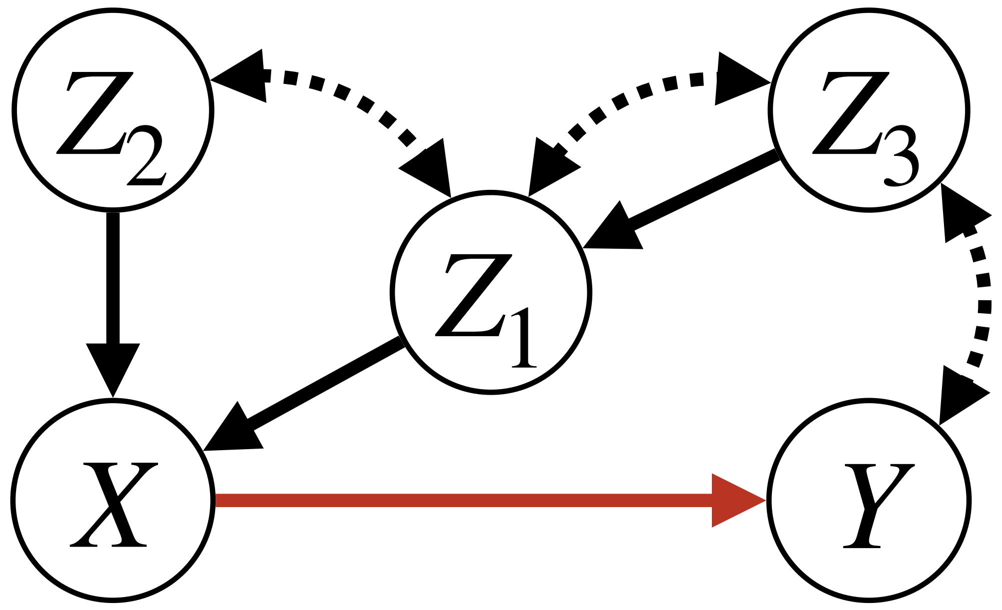
$\mathbf{Y} = \{Y\}$
$\mathbf{Pa_X} = \{Z_1, Z_2\}$



$Pa_x = \{Z_1, Z_2\}$

$$P(y \mid do(x)) = \sum_{z_1, z_2} P\left(y \mid x, z_1, z_2\right) P\left(z_1, z_2\right)$$

After conditioning on the parents, the association between X and Y is only due to the direct path.

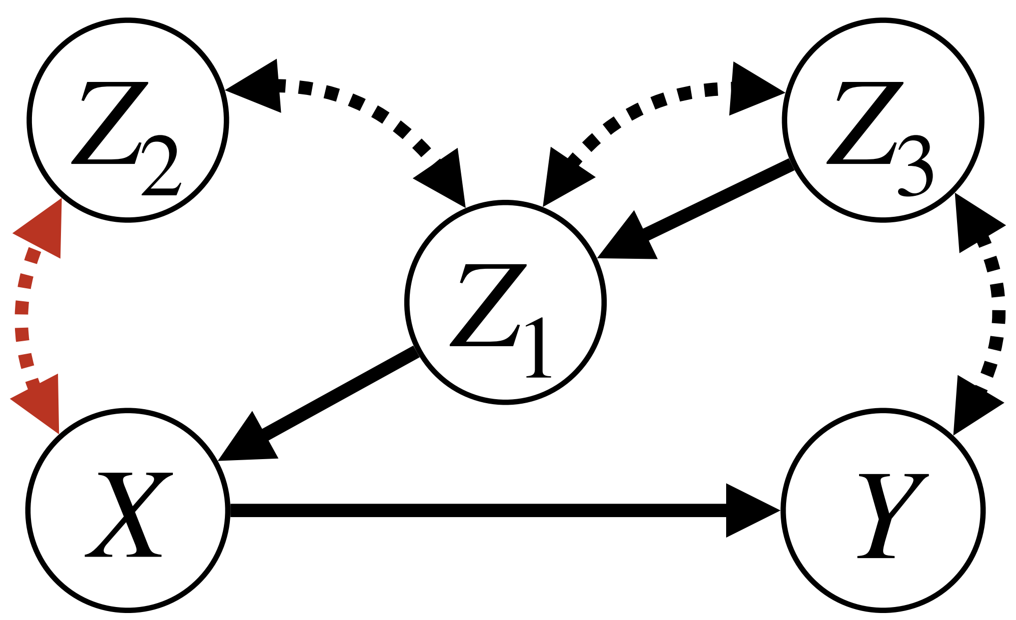
Identification in Semi-Markovian Models

Adjustment over parents:

Let G be a causal graph with **no unmeasured parents**.

Then, the effect of $\mathbf{X}$ on $\mathbf{Y}$ is given by:

$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{pa_x}} P\left(\mathbf{y} \mid \mathbf{x}, \mathbf{pa_x}\right) P\left(\mathbf{pa_x}\right)$$



$$Pa_x = \{Z_2\}$$
$$U_x = \{U_{X,Z2}\}$$

$$P(y \mid do(x)) = ?$$

Identification in Semi-Markovian Models

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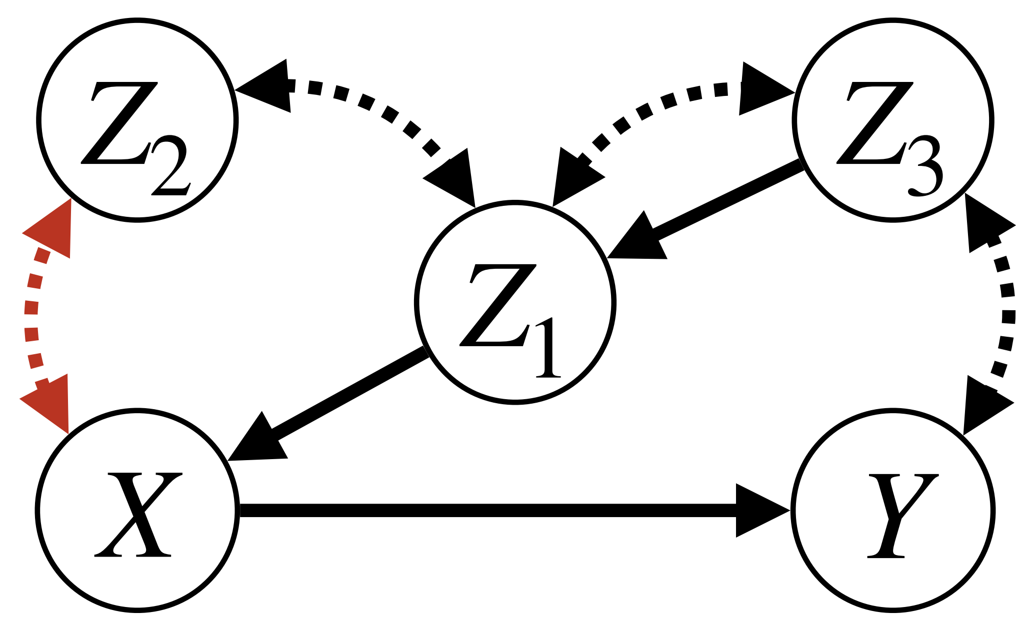

$$Pa_x = \{Z_2\}$$
$$U_x = \{U_{X,Z2}\}$$

$$P(y \mid do(x)) = \sum_{z_1, z_2} P\left(y \mid x, z_1, z_2\right) P\left(z_1, z_2\right)$$

Identification in Semi-Markovian Models

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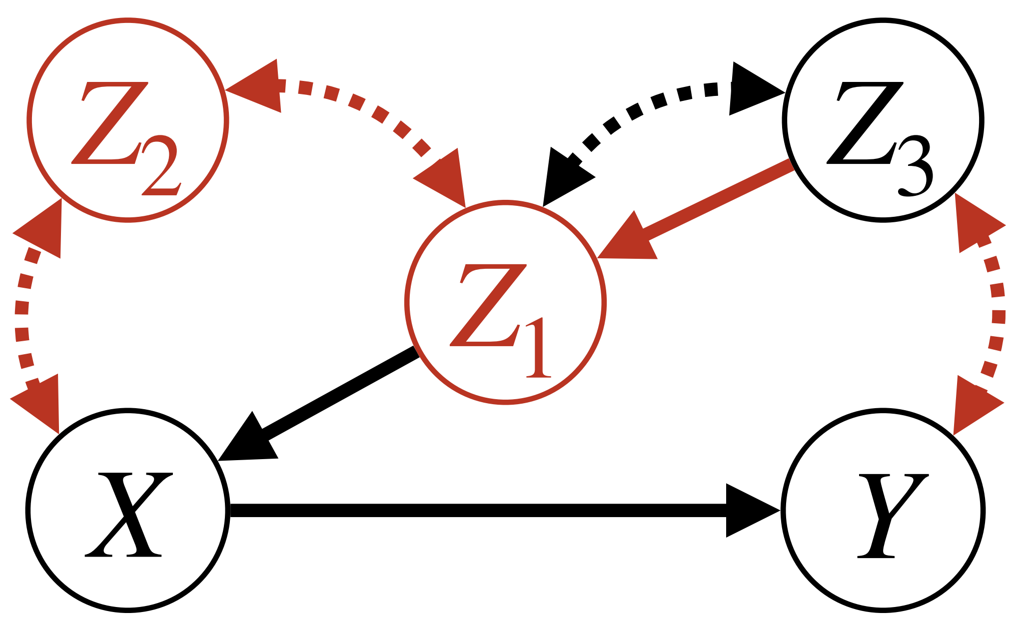


$$Pa_x = \{Z_2\}$$
$$U_x = \{U_{X,Z2}\}$$

$$P(y \mid do(x)) = \sum_{z_1, z_2} P\left(y \mid x, z_1, z_2\right) P\left(z_1, z_2\right)$$ ✗

After conditioning on the $\{Z_1, Z_2\}$, the association between X and Y is also due to a spurious / confounding path.

Identification via Backdoor Criterion

In $G_{\underline{\mathbf{X}}}$, all non-backdoor paths are severed

Let $\mathbf{X}$ be a set of treatment variables and $\mathbf{Y}$ a set of outcome variables in the causal graph G.

If there exists a set $\mathbf{Z}$ such that:

1. $\mathbf{Z}$ d-separates $\mathbf{X}$ and $\mathbf{Y}$ in the graph $G_{\underline{\mathbf{X}}}$, i.e., the graph resulting from cutting the arrows out of $\mathbf{X}$

2. no node in $\mathbf{Z}$ is a descendant of a variable $X \in \mathbf{X}$ in G (all variables in $\mathbf{Z}$ are pre-treatment)

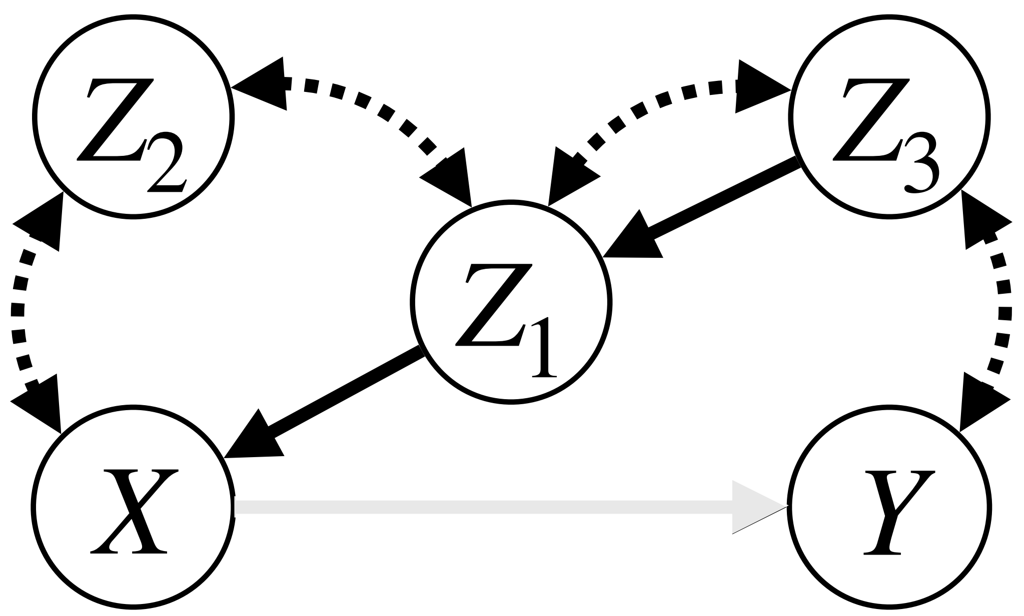
Then, $\mathbf{Z}$ satisfies the ***backdoor criterion*** for $(\mathbf{X}, \mathbf{Y})$ and, then the effect of $\mathbf{X}$ on $\mathbf{Y}$ is given by:

$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{z}} P\left(\mathbf{y} \mid \mathbf{x}, \mathbf{z}\right) P\left(\mathbf{z}\right)$$

$\mathbf{Z}$, a set of covariates, admissible for backdoor adjustment

$\mathbf{X} = \{X\}$

$\mathbf{Y} = \{Y\}$



$\mathbf{Z} = \{Z_1\}$

$\mathbf{Z} = \{Z_1, Z_3\}$

Judea Pearl. Comment: Graphical models, causality and intervention. Stat. Sci., 8:266–269, 1993.

Identification via Backdoor Criterion

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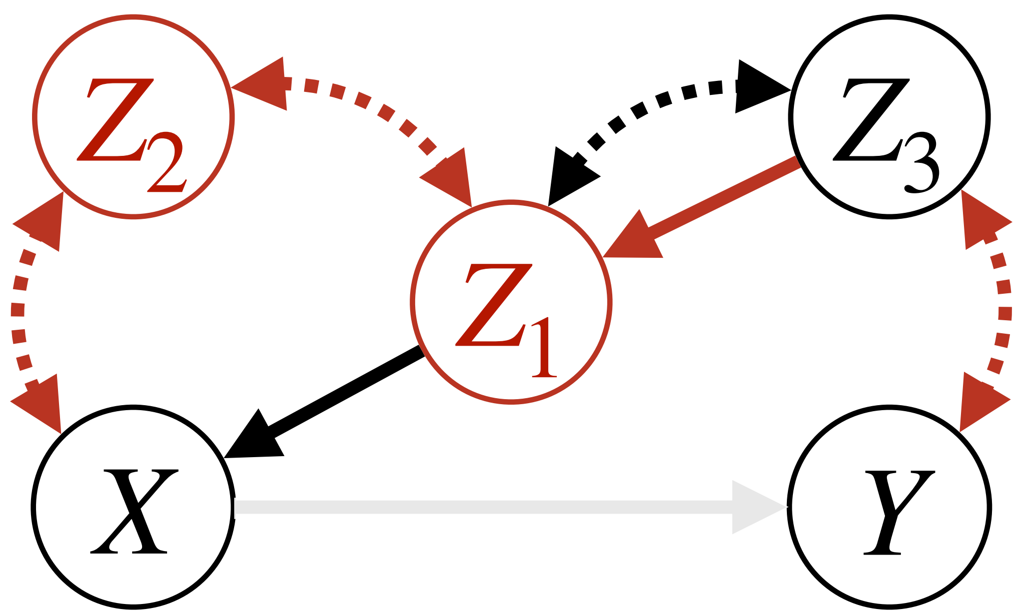
$\mathbf{Z}$, a set of covariates, admissible for backdoor adjustment

$\mathbf{X} = \{X\}$
$\mathbf{Y} = \{Y\}$



$\mathbf{Z} = \{Z_1\}$

$\mathbf{Z} = \{Z_1, Z_3\}$

$\mathbf{Z} = \{Z_1, Z_2\}$ ✘

Judea Pearl. Comment: Graphical models, causality and intervention. Stat. Sci., 8:266–269, 1993.

Identification via Backdoor Criterion

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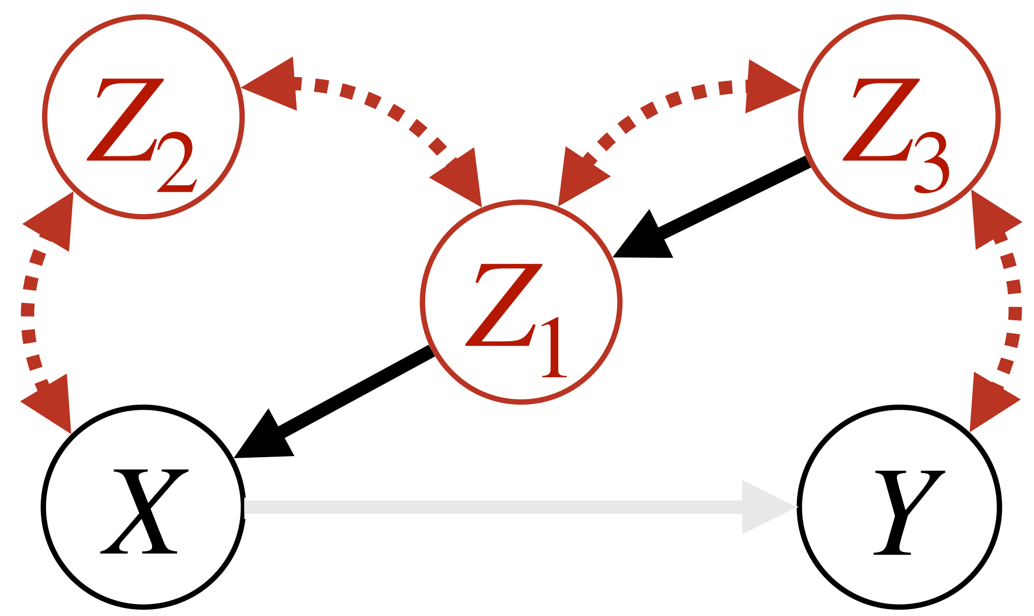
$$P(\mathbf{y} \mid do(\mathbf{x})) = \sum_{\mathbf{z}} P\left(\mathbf{y} \mid \mathbf{x}, \mathbf{z}\right) P\left(\mathbf{z}\right)$$

$\mathbf{Z}$, a set of covariates, admissible for backdoor adjustment

$\mathbf{X} = \{X\}$
$\mathbf{Y} = \{Y\}$



$\mathbf{Z} = \{Z_1\}$

$\mathbf{Z} = \{Z_1, Z_3\}$

$\mathbf{Z} = \{Z_1, Z_2, Z_3\}$ ✗

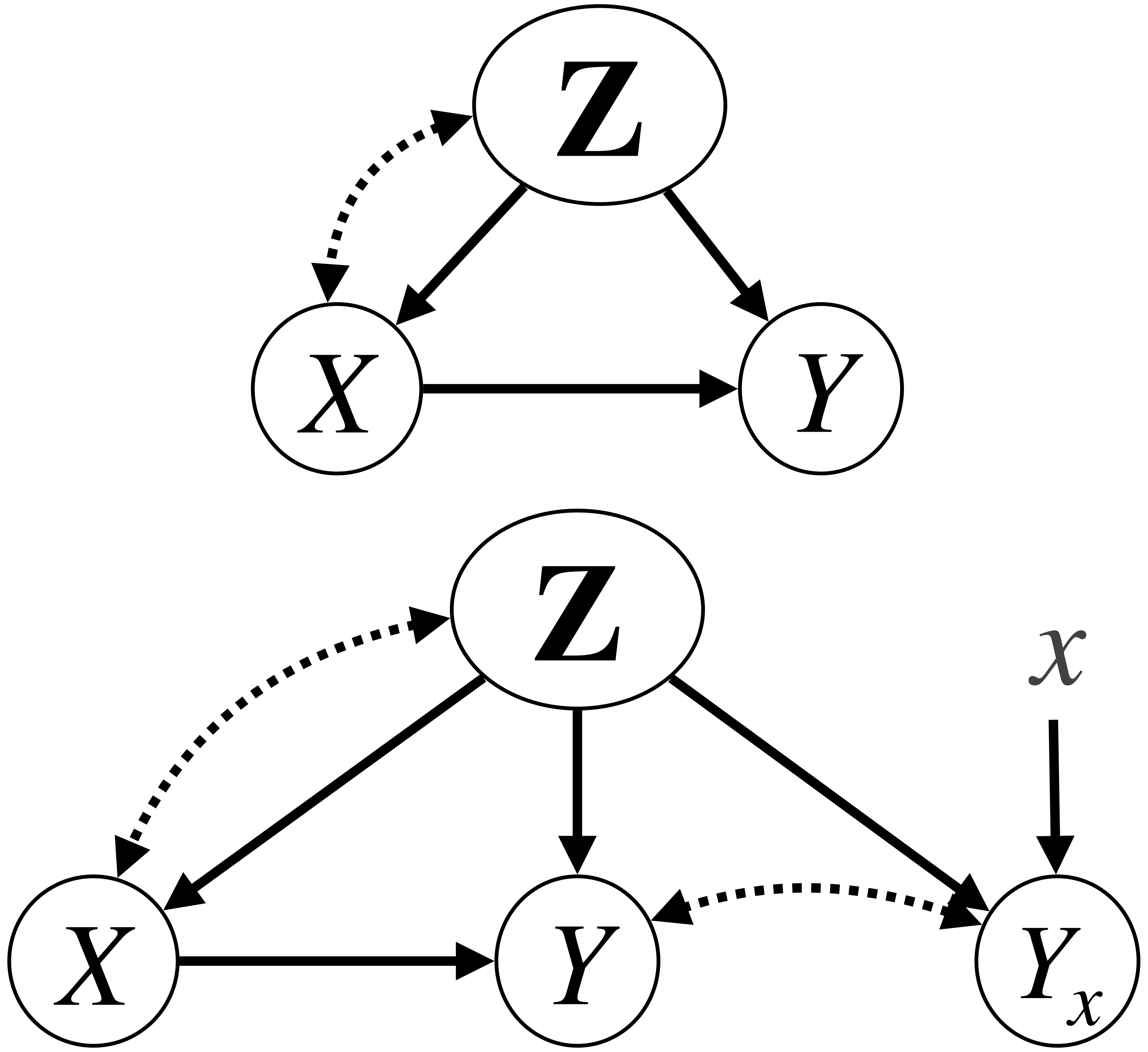
Judea Pearl. Comment: Graphical models, causality and intervention. Stat. Sci., 8:266–269, 1993.

Counterfactual Interpretation of Backdoor

Theorem 4.3.1, Pearl's Primer Book

Theorem: If a set $\mathbf{Z}$ satisfies the *backdoor criterion* w.r.t. the ordered pair (X, Y), then, for all x, it holds that $Y_x \perp\!\!\!\perp X | \mathbf{Z}$.

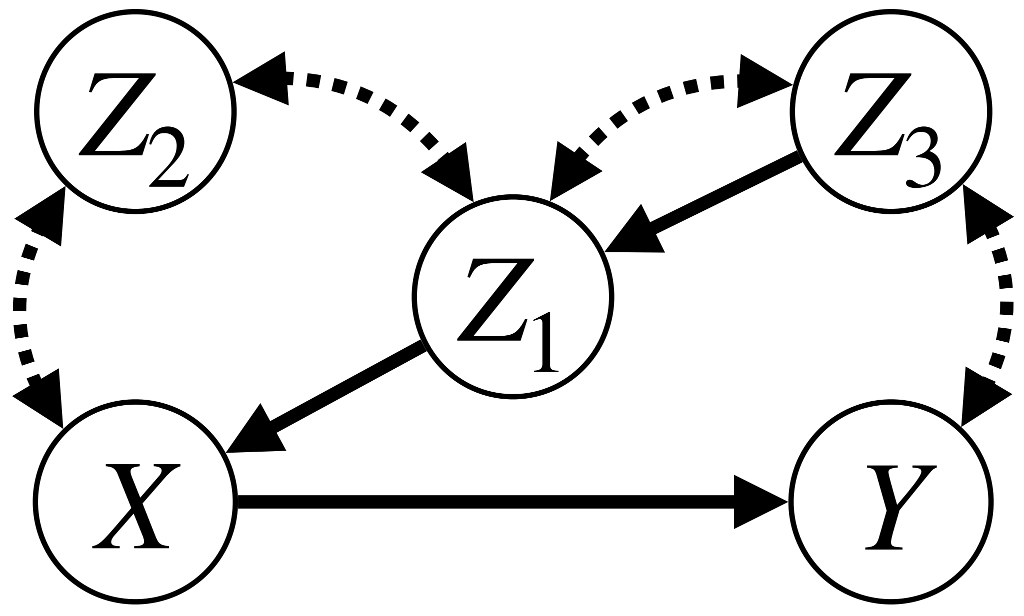
Although the satisfiability of $\mathbf{Z}$ to the *backdoor criterion* can be tested given a causal diagram or a PAG, the condition $Y_x \perp\!\!\!\perp X | \mathbf{Z}$ is sometimes framed as an assumption, referred to as **(conditional) ignorability**, **exchangeability** or **unconfoundedness**.



$Y_x \perp\!\!\!\perp X | Z$

Estimation via Propensity Scores

Consider the case in which the causal effect of X on Y is identifiable through adjustment over a set of variables $\mathbf{Z}$, i.e.,



$$P(y \mid do(x)) = \sum_{\mathbf{z}} P(y \mid x, \mathbf{z}) P(\mathbf{z})$$

$$= \sum_{\mathbf{z}} \frac{P(y \mid x, \mathbf{z}) P(x \mid \mathbf{z}) P(\mathbf{z})}{P(x \mid \mathbf{z})}$$

$$= \sum_{\mathbf{z}} \frac{P(y, x, \mathbf{z})}{P(x \mid \mathbf{z})}$$

$\mathbf{Z} = \{Z_1\}$

$\mathbf{Z} = \{Z_1, Z_3\}$

Only if $\mathbf{Z}$ is admissible for adjustment, Propensity Score can be used to estimate $P(y \mid do(x))$.

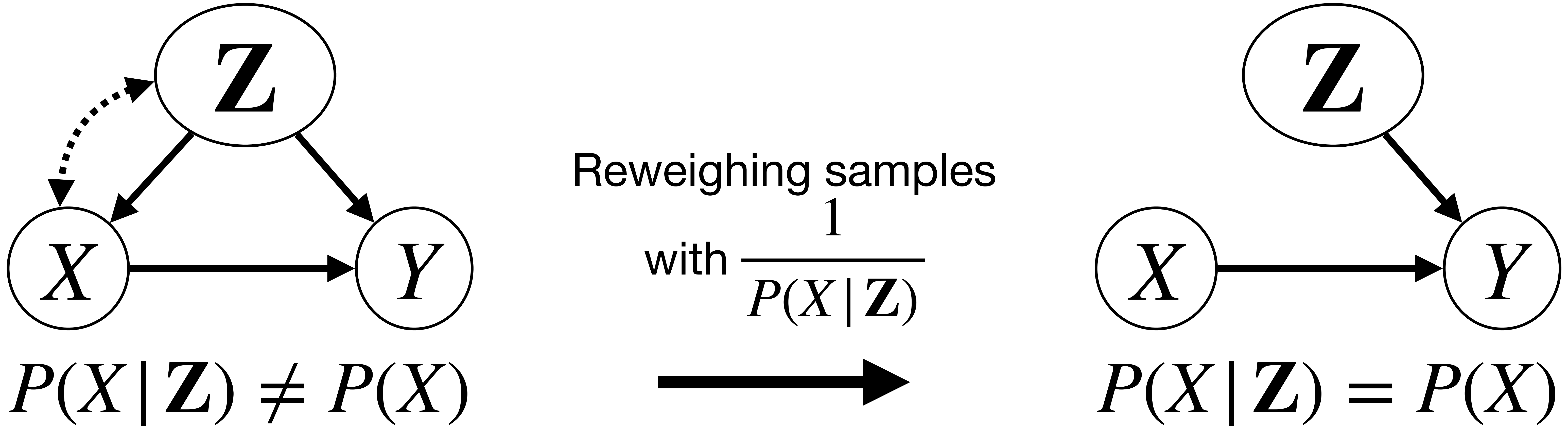
For X is binary/categorial: logistic/multinomial regression or ML-based classification

For X continuous: ML-based regression techniques.

The interventional joint distribution can be easily derived by reweighing the observational joint distribution with the inverse of the propensity score!

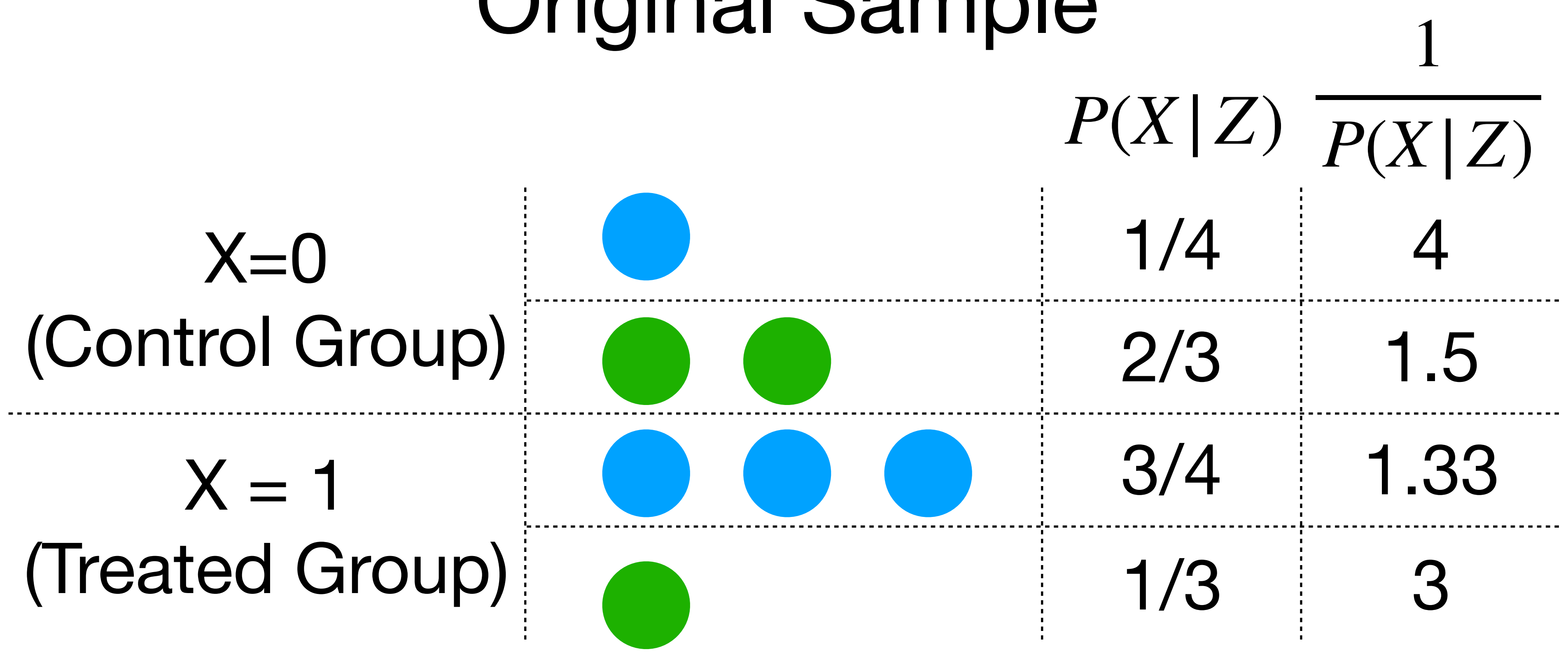
Inverse Probability Weighting

After reweighing the observational samples, we obtain *pseudo* interventional samples:

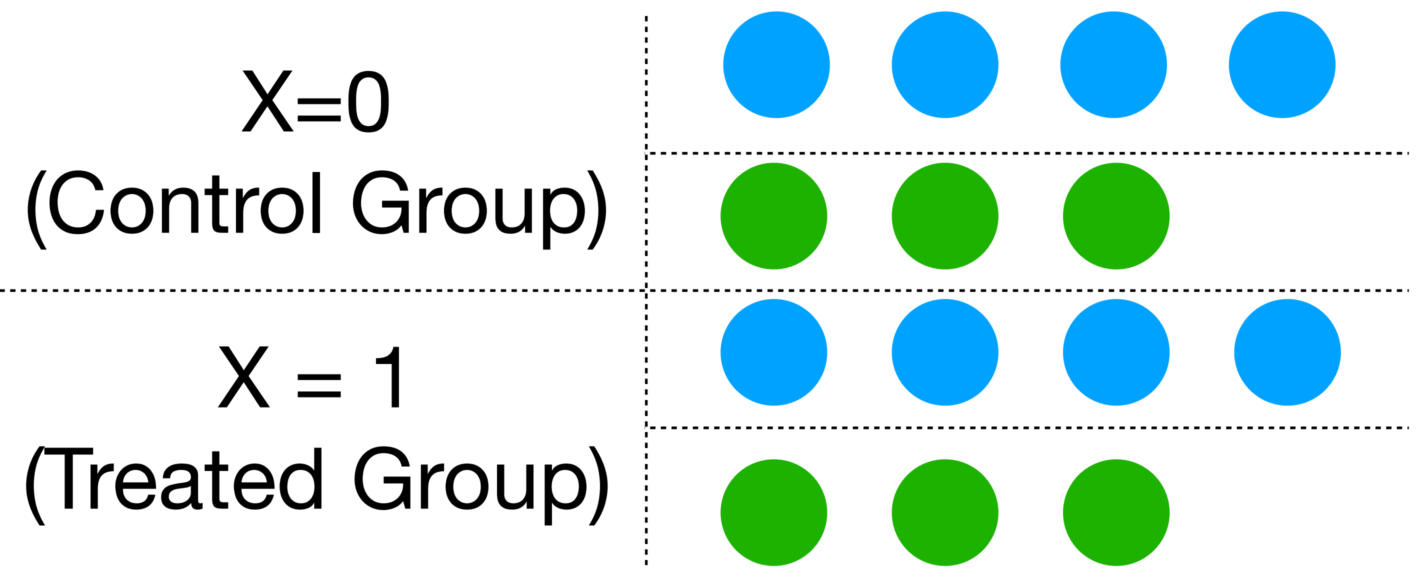


Original Sample

Pseudo interventional Sample



Imbalanced



Balanced

Inverse Probability Weighting

This gives us the following estimator of $E(Y|do(x))$, from a sample $\{x_i, y_i, \mathbf{z_i}\}_{i=1}^{N}$:

$$\hat{E}(Y|do(x)) = \frac{1}{N}\sum_{i=1}^{N}\frac{y_i \mathbf{1}_{\{x_i=x\}}}{\hat{P}(x_i|\mathbf{z}_i)}$$

The mean of all values y_i, inversely weighted according to the propensity score.

The Average Treatment Effect (ATE) of a binary treatment can be estimated as:

$$\hat{E}(Y|do(X=1)) - \hat{E}(Y|do(X=0))$$

$$= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_i \mathbf{1}_{\{x_i=1\}}}{\hat{P}(X=1|\mathbf{z}_i)} - \frac{y_i \mathbf{1}_{\{x_i=0\}}}{\hat{P}(X=0|\mathbf{z}_i)}\right)$$

What if backdoor adjustment does not work?

Identification via Front-Door Adjustment

Let $\mathbf{X}$ be a set of treatment variables and $\mathbf{Y}$ a set of outcome variables in the causal graph G.

If there exists a set $\mathbf{M}$ such that:

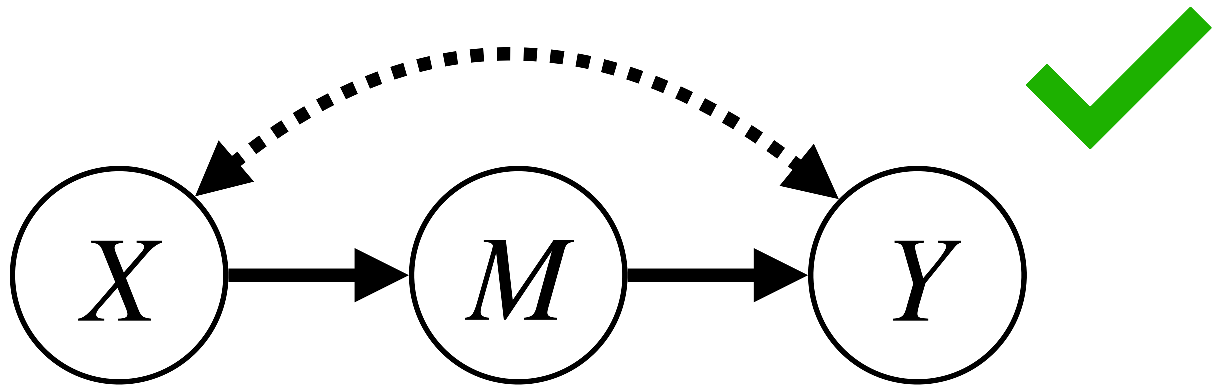
1. $\mathbf{M}$ intercepts all directed paths from any vertex $X \in \mathbf{X}$ to any vertex $Y \in \mathbf{Y}$;
2. There is no unblocked back-door path from any vertex $X \in \mathbf{X}$ to vertex $M \in \mathbf{M}$; and
3. All back-door paths from any vertex $M \in \mathbf{M}$ to any vertex $Y \in \mathbf{Y}$ are blocked by $\mathbf{X}$.

Then, $\mathbf{M}$ satisfies the ***front-door criterion*** and, then the effect of $\mathbf{X}$ on $\mathbf{Y}$ is given by:

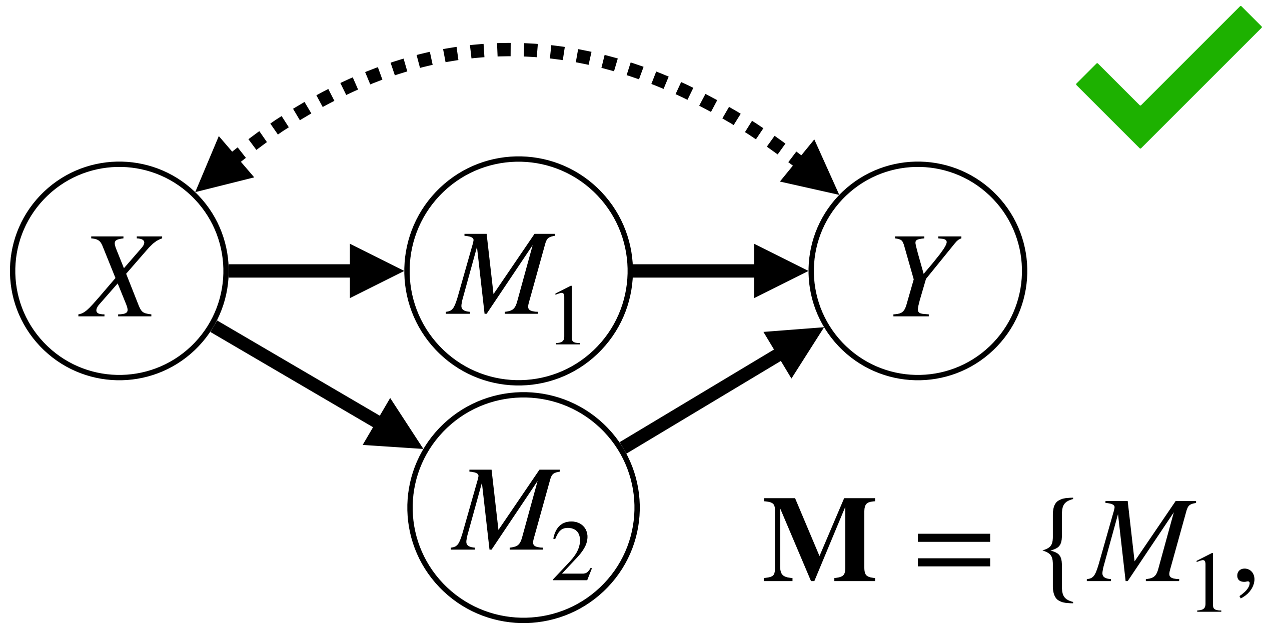
$$P(\mathbf{y} \,|\, do(\mathbf{x})) = \sum_{\mathbf{m}} P(\mathbf{m} \,|\, \mathbf{x}) \sum_{\mathbf{x}'} P(\mathbf{y} \,|\, \mathbf{m}, \mathbf{x}') P(\mathbf{x}')$$

$\mathbf{X} = \{X\}$

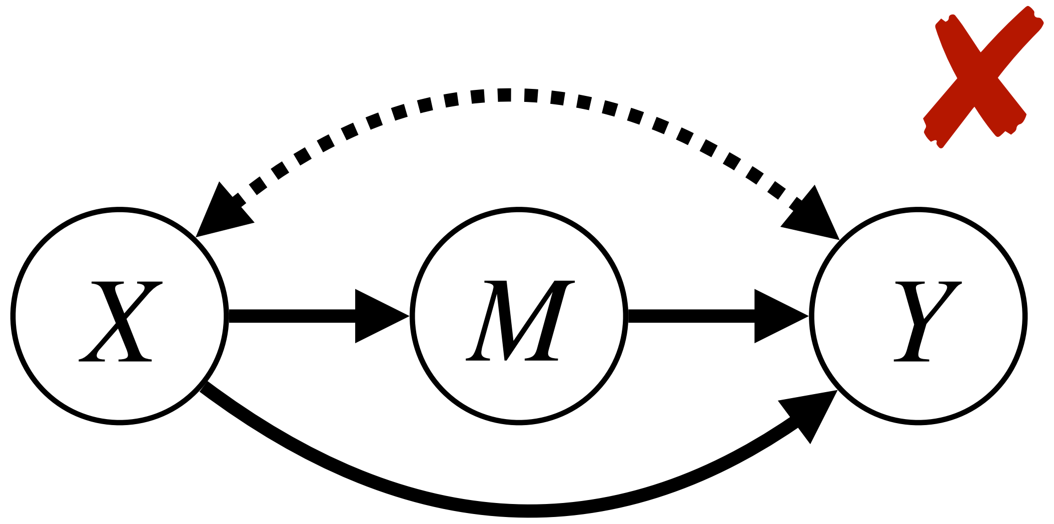
$\mathbf{Y} = \{Y\}$

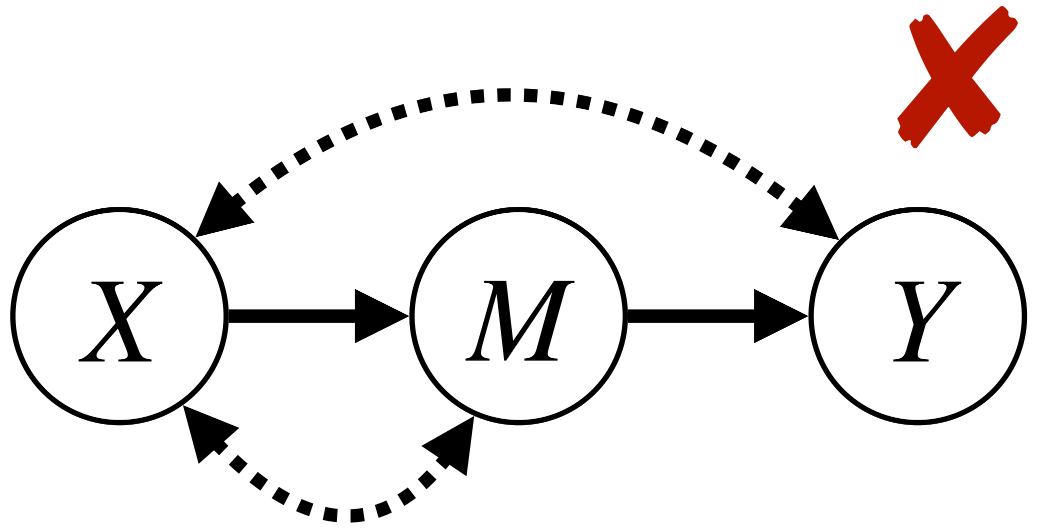


$\mathbf{M} = \{M\}$

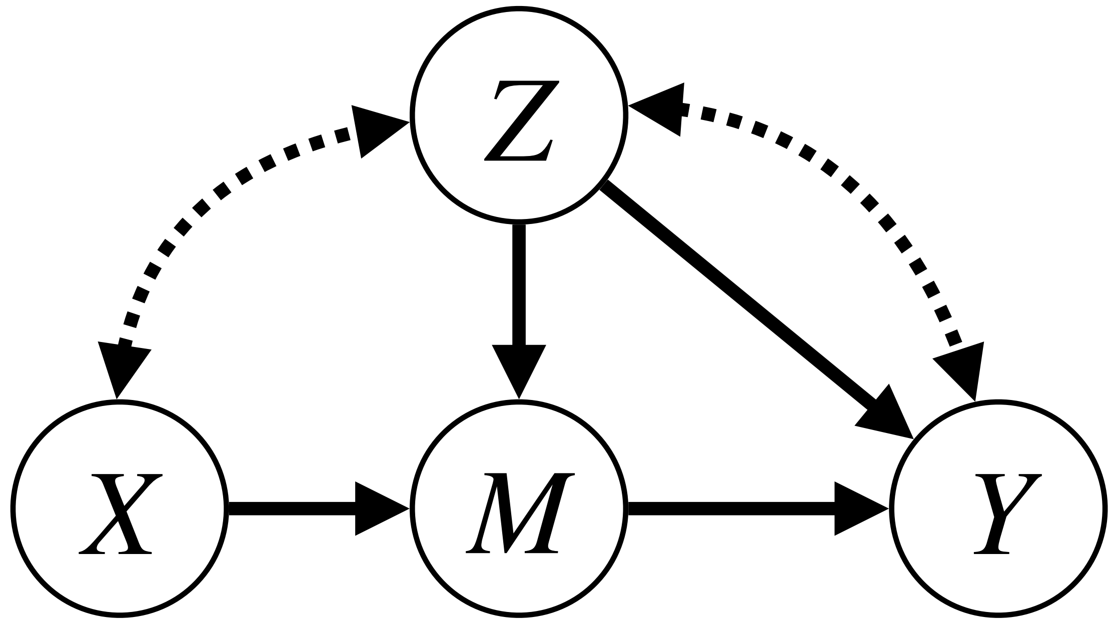


$\mathbf{M} = \{M_1, M_2\}$



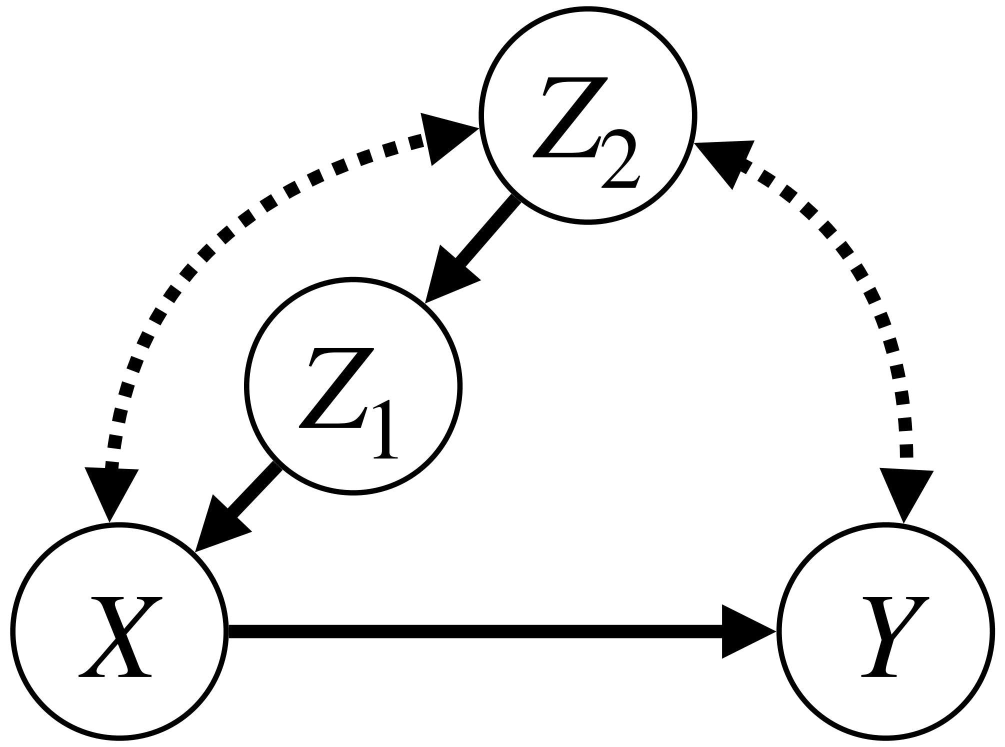


Many scenarios beyond back-door and front-door!



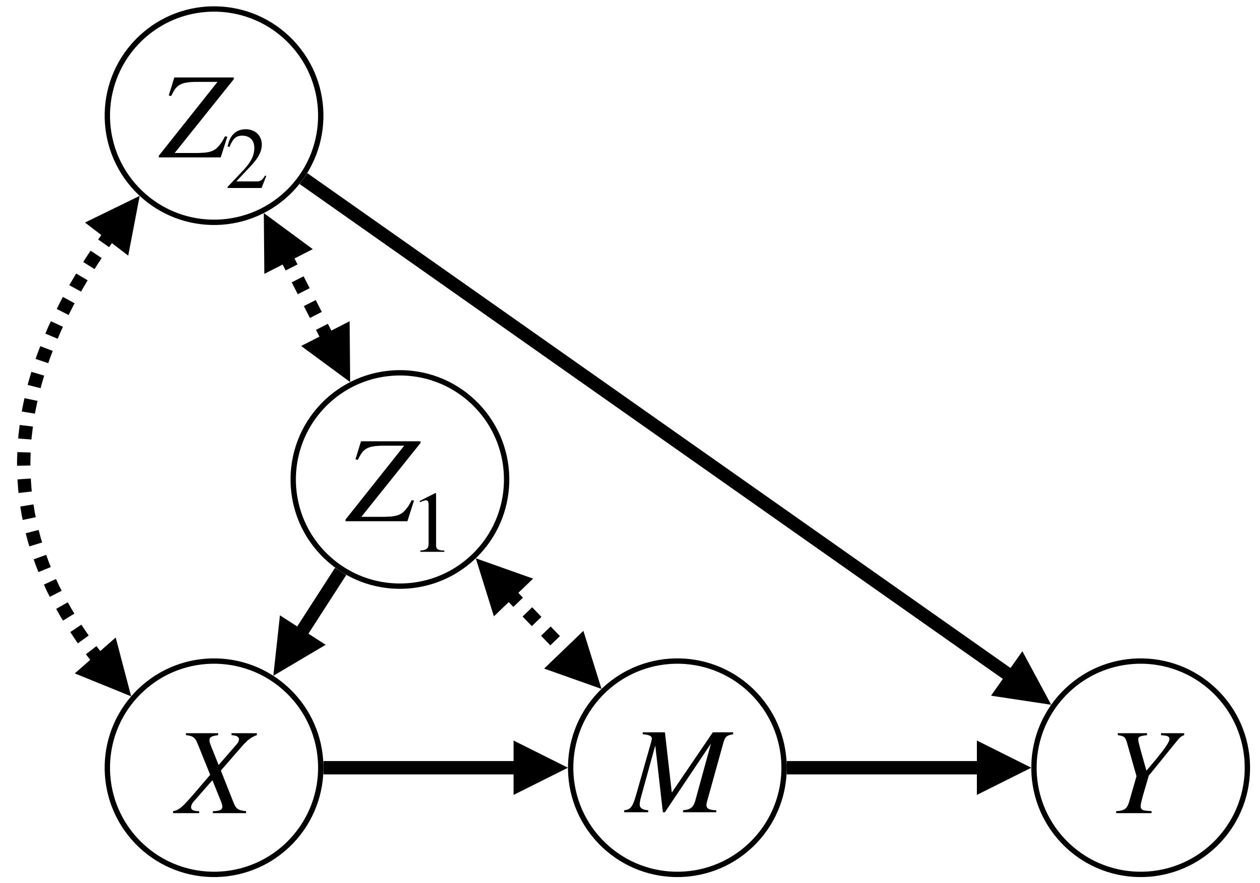
Conditional Front-Door

$$P(y \mid do(x)) = \sum_{m,z} P(m \mid x, z) \sum_{x'} P(y \mid m, x', z) P(x', z)$$



Napkin

$$P(y \mid do(x)) = \frac{\sum_{z_2} P(x, y \mid z_1, z_2) P(z_2)}{\sum_{z_2} P(x \mid z_1, z_2) P(z_2)}$$

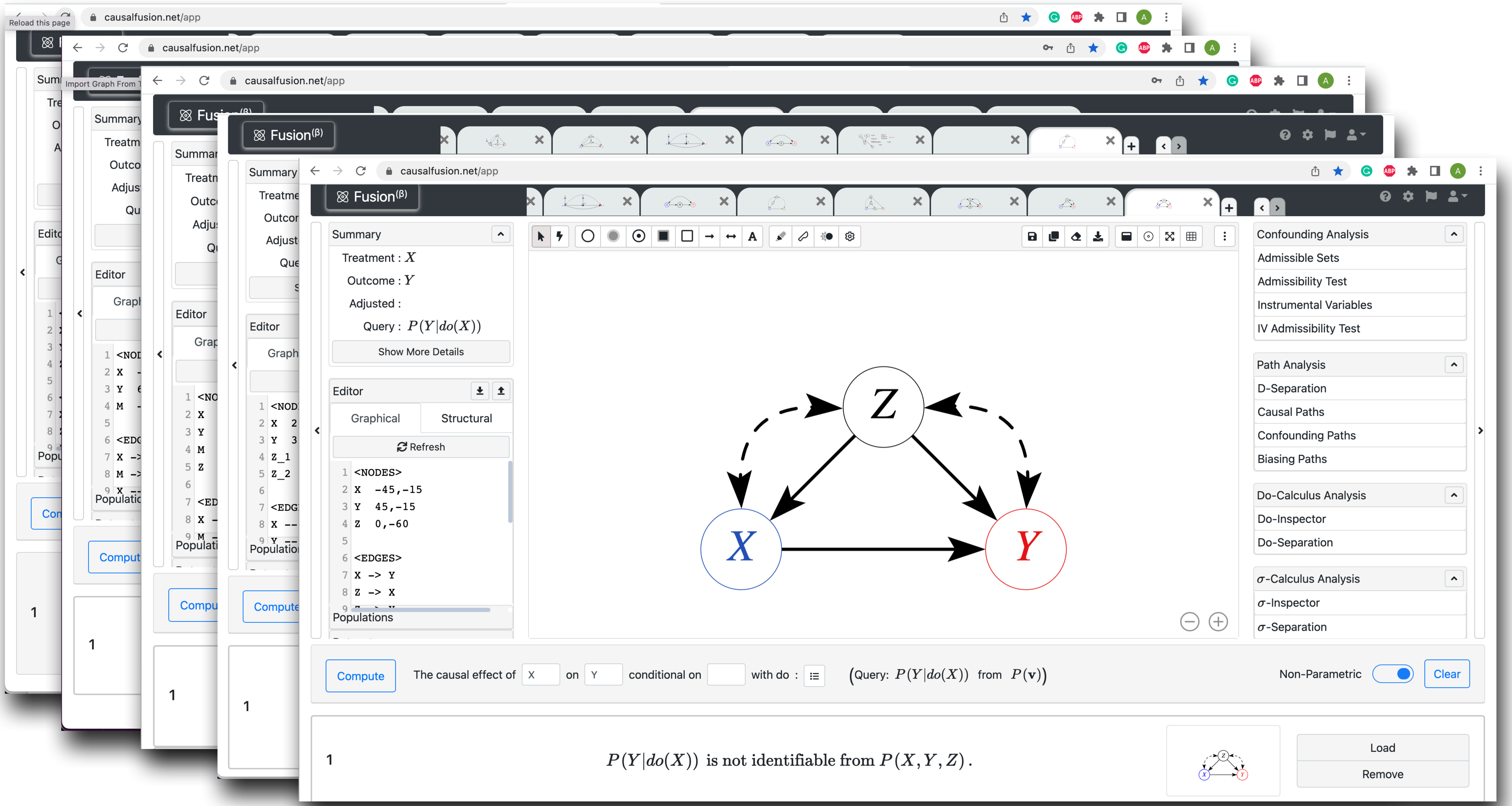


Unnamed

$$P(y \mid do(x)) = \sum_{z_2, z_3} P(y \mid x, z_1, z_2, z_3) P(z_2) \sum_{z_1} P(z_3 \mid x, z_1) P(z_1)$$

And many others….

http://causalfusion.net



Do-Calculus (a.k.a. Causal Calculus)

Pearl, 1995

Graphical conditions implying invariances between observational ($\mathscr{L}_1$) and interventional ($\mathscr{L}_2$) distributions

Theorem: Let $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}$ be any disjoint subjects of variables.

Rule 1 (Insertion/Deletion of Observations)

$$P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{x}, \mathbf{z}) = P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{z}), \text{ if } (\mathbf{Y} \perp\!\!\!\perp \mathbf{X} \mid \mathbf{Z}, \mathbf{W})_{G_{\overline{\mathbf{W}}}}$$

Rule 2 (Exchange of Actions and Observations)

$$P(\mathbf{y} \mid do(\mathbf{w}), do(\mathbf{x}), \mathbf{z}) = P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{x}, \mathbf{z}), \text{ if } (\mathbf{Y} \perp\!\!\!\perp \mathbf{X} \mid \mathbf{Z}, \mathbf{W})_{G_{\overline{\mathbf{W}}\underline{\mathbf{X}}}}$$

Rule 3 (Insertion/Deletion of Actions)

$$P(\mathbf{y} \mid do(\mathbf{w}), do(\mathbf{x}), \mathbf{z}) = P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{z}), \text{ if } (\mathbf{Y} \perp\!\!\!\perp \mathbf{X} \mid \mathbf{Z}, \mathbf{W})_{G_{\overline{\mathbf{W}}, \overline{\mathbf{X}(\mathbf{Z})}}}$$

$G_{\overline{\mathbf{W}}\underline{\mathbf{X}}}$: graph G after removing the incoming arrows into $\mathbf{W}$ and the outgoing arrows from $\mathbf{X}$;
$\mathbf{X}(\mathbf{Z})$: set of $\mathbf{X}$-nodes that are not ancestors of any $\mathbf{Z}$-node in $G_{\overline{\mathbf{W}}}$.

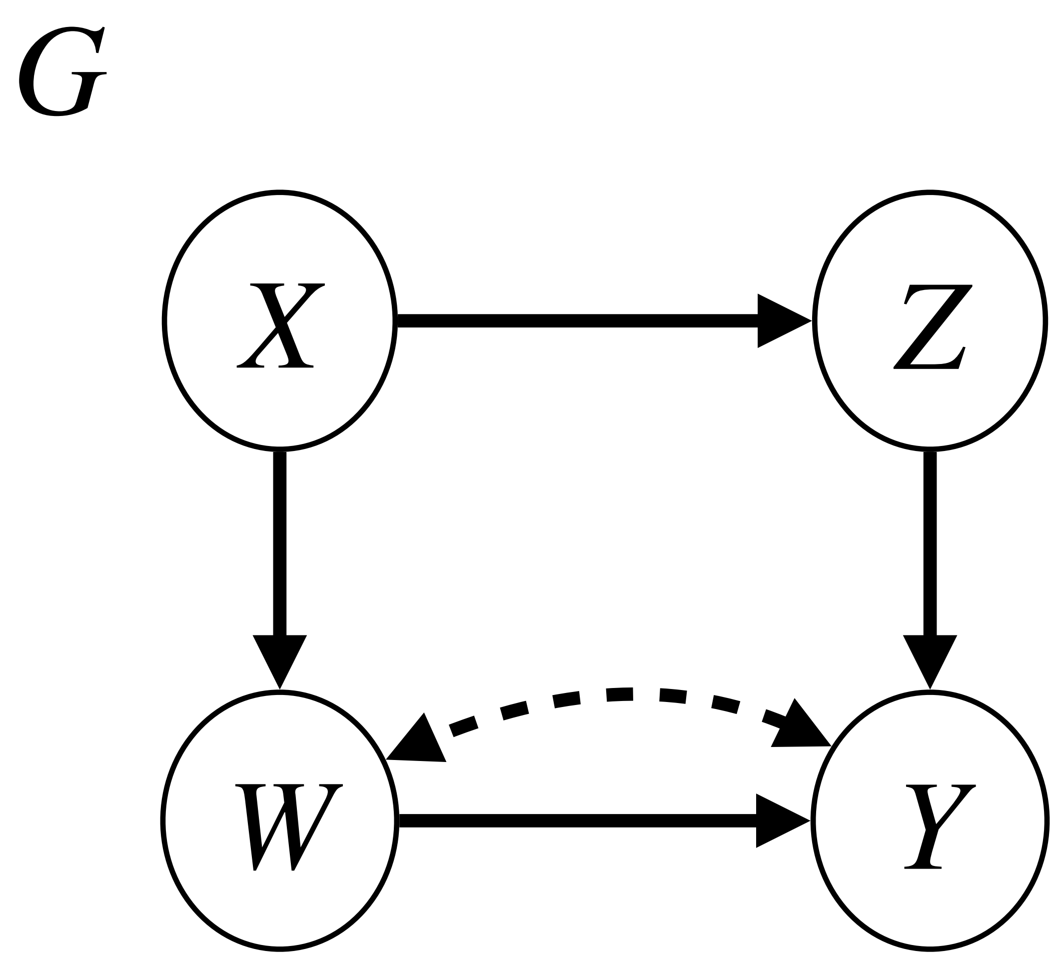
Do-Calculus - Rule 1

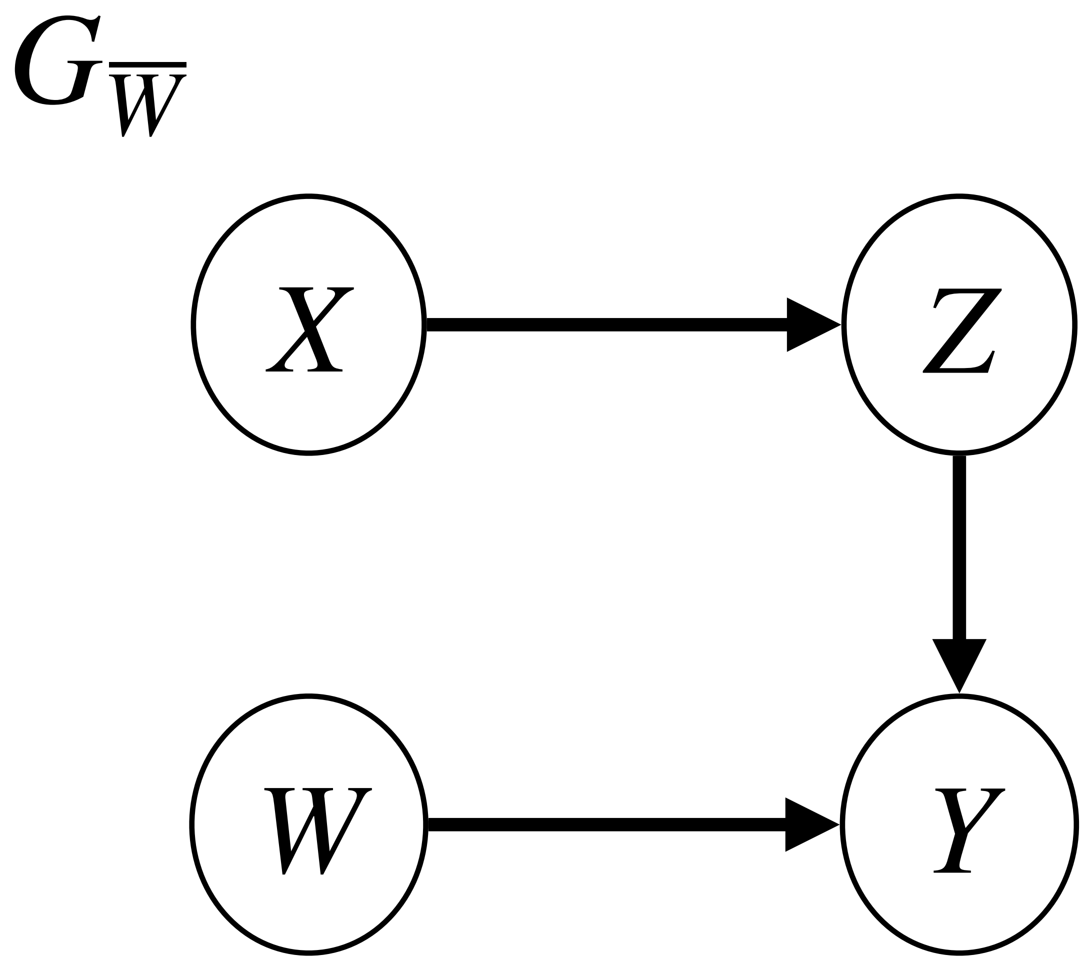
Theorem: Let $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}$ be any disjoint subjects of variables.

Rule 1 (Insertion/Deletion of Observations)

$$P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{x}, \mathbf{z}) = P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{z}), \text{ if } (\mathbf{Y} \perp\!\!\!\perp \mathbf{X} \mid \mathbf{Z}, \mathbf{W})_{G_{\overline{\mathbf{W}}}}$$

$\mathbf{X}$ is conditionally independent of $\mathbf{Y}$ given $\mathbf{Z} \cup \mathbf{W}$ in the interventional model $G_{\overline{\mathbf{W}}}$





$$(Y \perp\!\!\!\perp X \mid Z, W)_{G_{\overline{W}}}$$

$$\Longrightarrow \quad P(y \mid do(w), x, z) = P(y \mid do(w), z)$$

Do-Calculus - Rule 2

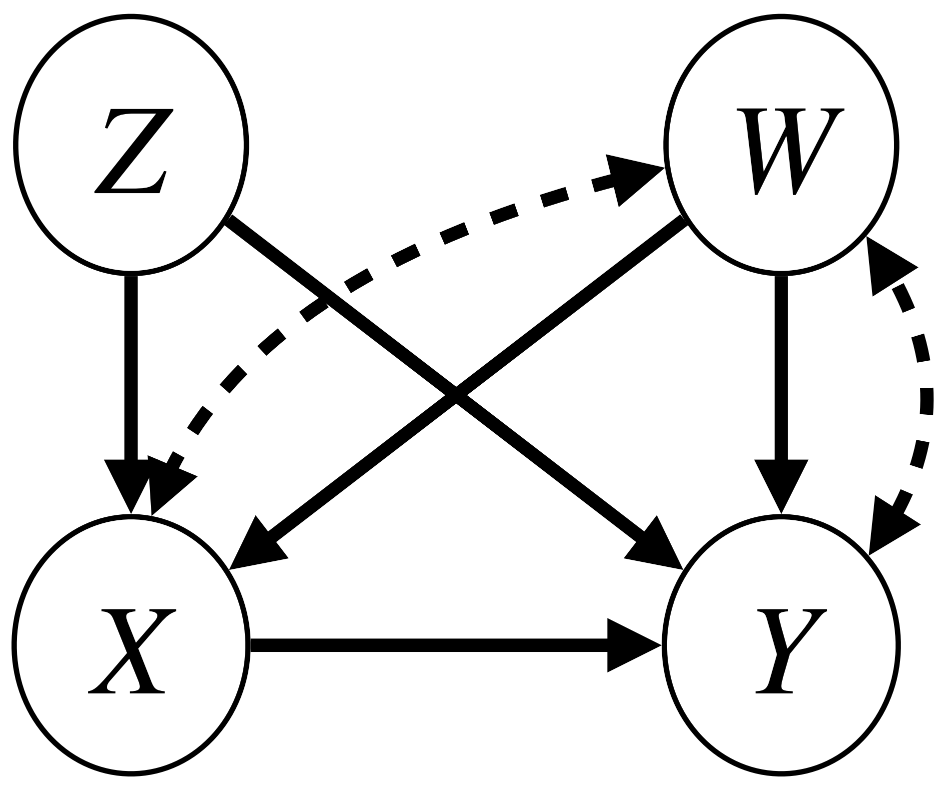
Theorem: Let $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}$ be any disjoint subjects of variables.

Rule 2 (Exchange of Actions and Observations)

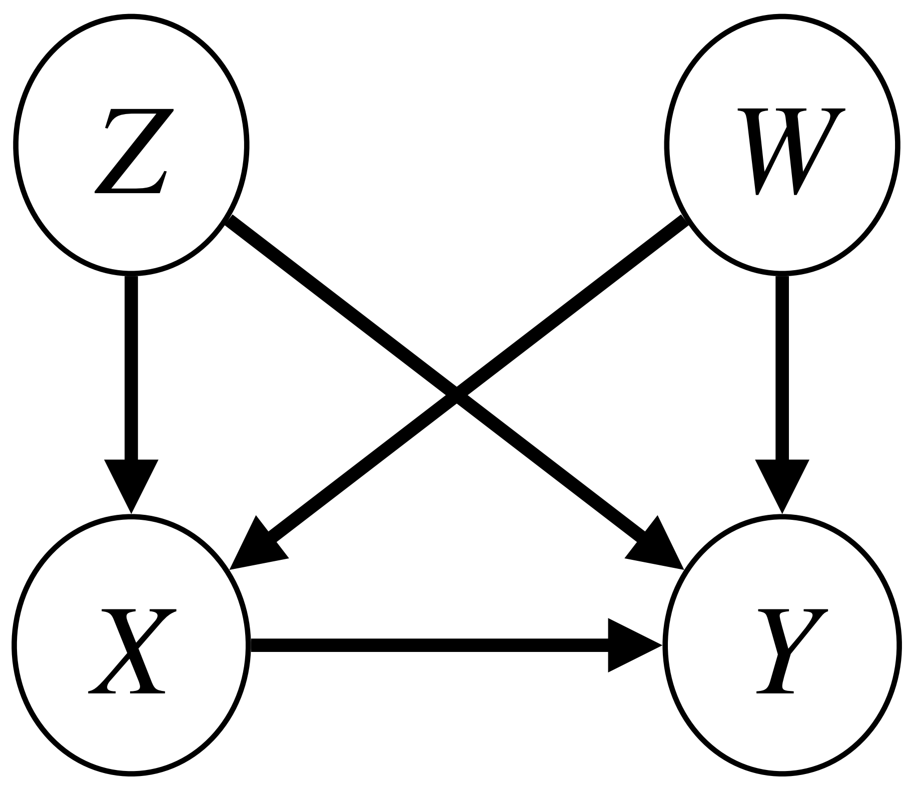
$$P(\mathbf{y} \mid do(\mathbf{w}), do(\mathbf{x}), \mathbf{z}) = P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{x}, \mathbf{z}), \text{ if } (\mathbf{Y} \perp\!\!\!\perp \mathbf{X} \mid \mathbf{Z}, \mathbf{W})_{G_{\overline{\mathbf{W}}\underline{\mathbf{X}}}}$$

$\mathbf{X}$ and $\mathbf{Y}$ are unconfounded given $\mathbf{Z} \cup \mathbf{W}$ in the interventional model $G_{\overline{\mathbf{W}}}$

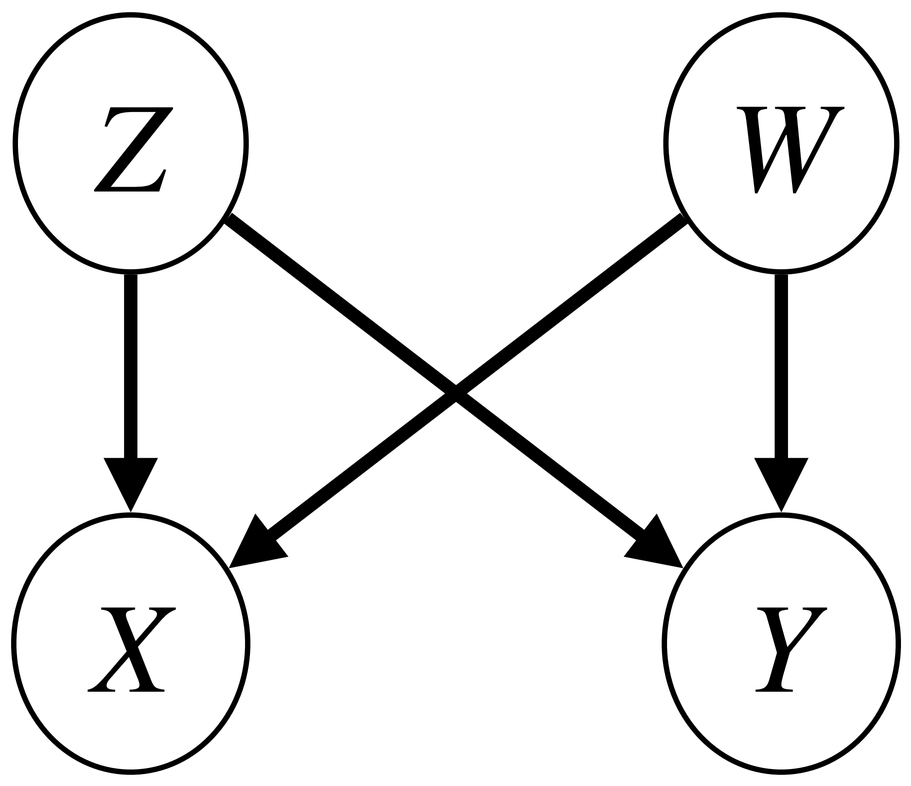
G



$G_{\overline{W}}$



$G_{\overline{W}\underline{X}}$



$$(X \perp\!\!\!\perp Y \mid Z, W)_{G_{\overline{W}\underline{X}}}$$

$$\implies P(y \mid do(w), do(x), z) = P(y \mid do(w), x, z)$$

Do-Calculus - Rule 3

Theorem: Let $\mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{W}$ be any disjoint subjects of variables.

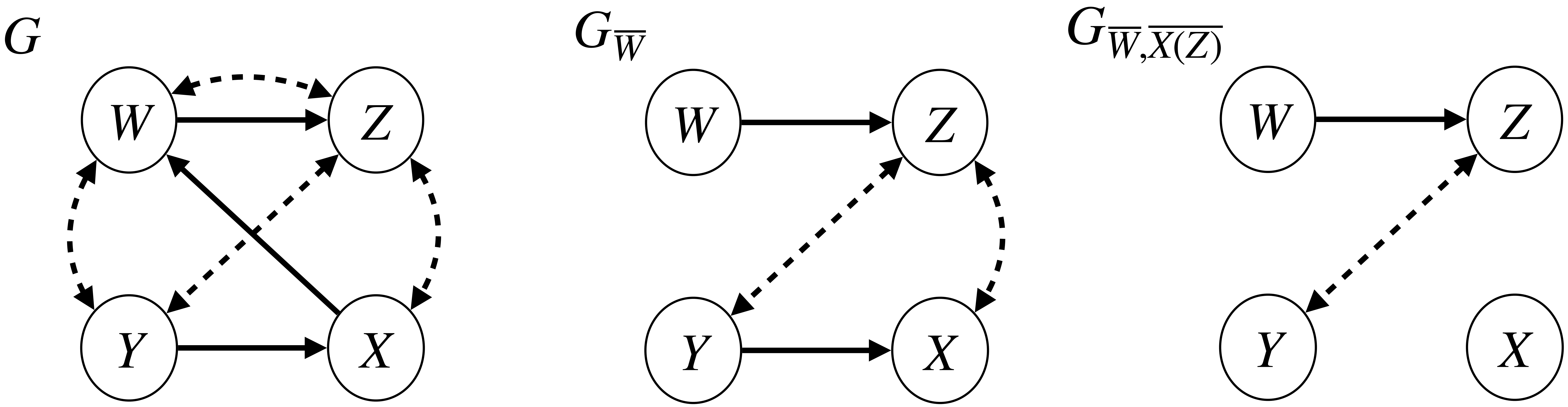
Rule 3 (Insertion/Deletion of Actions)

$$P(\mathbf{y} \mid do(\mathbf{w}), do(\mathbf{x}), \mathbf{z}) = P(\mathbf{y} \mid do(\mathbf{w}), \mathbf{z}), \text{ if } (\mathbf{Y} \perp\!\!\!\perp \mathbf{X} \mid \mathbf{Z}, \mathbf{W})_{G_{\overline{\mathbf{W}}, \overline{\mathbf{X}(\mathbf{Z})}}}$$

$\mathbf{Y}$ is not affected by any action on $\mathbf{X}$ given $\mathbf{Z} \cup \mathbf{W}$ in the interventional model $G_{\overline{\mathbf{W}}}$

$G_{\overline{\mathbf{W}}\underline{\mathbf{X}}}$: graph G after removing the incoming arrows into $\mathbf{W}$ and the outgoing arrows from $\mathbf{X}$;

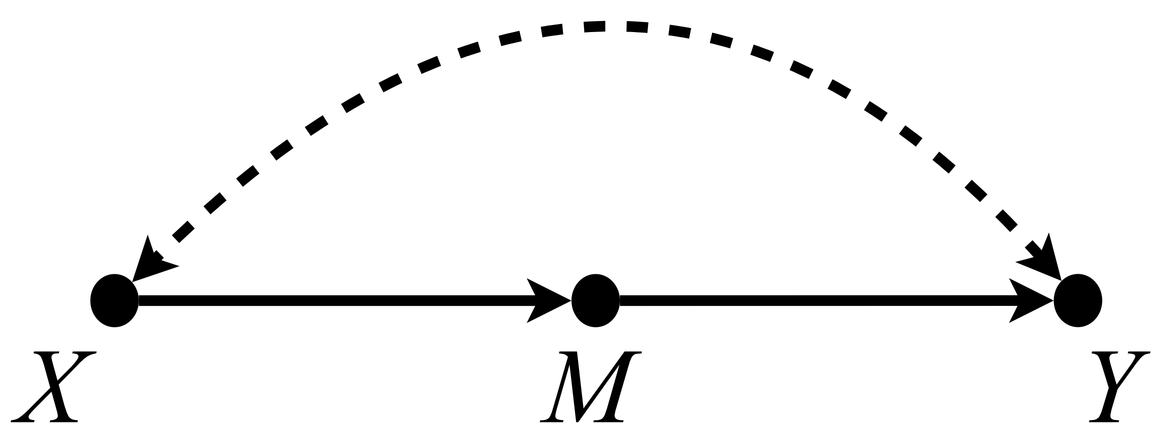
$\mathbf{X}(\mathbf{Z})$: set of $\mathbf{X}$-nodes that are not ancestors of any $\mathbf{Z}$-node in $G_{\overline{\mathbf{W}}}$.



$$(Y \perp\!\!\!\perp X \mid Z, W)_{G_{\overline{W}, \overline{X(Z)}}}$$

$$\Longrightarrow \quad P(y \mid do(w), do(x), z) = P(y \mid do(w), z)$$

Identification in Non-Markovian Models



$$P(y \mid do(x)) = \sum_m P(y \mid do(x), m) P(m \mid do(x))$$ Probability Axioms

$$= \sum_m P(y \mid do(x), do(m)) P(m \mid do(x))$$ Rule 2

$$= \sum_m P(y \mid do(x), do(m)) P(m \mid x)$$ Rule 2

$$= \sum_m P(y \mid do(m)) P(m \mid x)$$ Rule 3

$$= \sum_{x'} \sum_m P(y \mid do(m), x') P(x' \mid do(m)) P(m \mid x)$$ Probability Axioms

$$= \sum_{x'} \sum_m P(y \mid m, x') P(x' \mid do(m)) P(m \mid x)$$ Rule 2

$$= \sum_{x'} \sum_m P(y \mid m, x') P(x' \mid m) P(m \mid x)$$ Rule 3

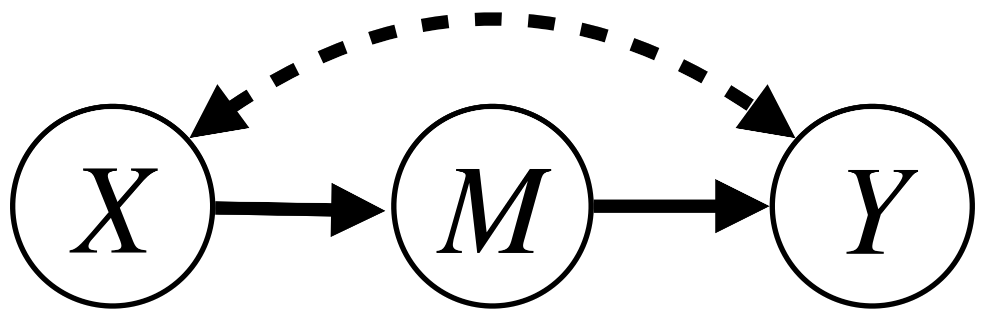
The Identify (ID) Algorithm

ID-Algorithm and many recent generalizations.

1 **Query**

$P(y \mid do(x))$

2 **Causal Contraints**



3 **Probability Distributions**

$P(x, m, y)$ **Observational Distribution**

Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_m P(m \mid x) \sum_{x'} P(y \mid m, x')P(x')$$

Interventional Distribution ← Available Distributions

👍

- Tian, J. and Pearl, J. A General Identification Condition for Causal Effects. In Proceedings of the Eighteenth National Conference on Artificial Intelligence (AAAI 2002), pp. 567–573, Menlo Park, CA, 2002. AAAI Press/MIT Press.

Advances on Effect Identification given a Causal Diagram

Identification from observational and experimental data:

Lee, S., Correa, J., and Bareinboim, E. (2019). General identifiability with arbitrary surrogate experiments. In *Proceedings of the 35th Conference on Uncertainty in Artificial Intelligence*, volume 35, Tel Aviv, Israel. AUAI Press.

J. Correa, S. Lee, E. Bareinboim. (2021) Nested Counterfactual Identification from Arbitrary Surrogate Experiments. In Proceedings of the 35th Annual Conference on Neural Information Processing Systems

Identification of stochastic/soft (and possibly imperfect) interventions:

Correa, J. and Bareinboim, E. (2020). A calculus for stochastic interventions: Causal effect identification and surrogate experiments. In *Proceedings of the 34th AAAI Conference on Artificial Intelligence*, New York, NY. AAAI Press.

Advances on Effect Identification given a Causal Diagram

Identification and Estimation via Deep Neural Networks:

Xia, K., Lee, K.-Z., Bengio, Y., and Bareinboim, E. (2021). The causal-neural connection: Expressiveness, learnability, and inference. *Advances in Neural Information Processing Systems*, 34.

Xia, K., Pan, Y.,and Bareinboim, E. (2023) Neural Causal Models for Counterfactual Identification and Estimation. In Proceedings of the 11th International Conference on Learning Representations.

Partial Effect Identification:

Kirtan Padh, Jakob Zeitler, David Watson, Matt Kusner, Ricardo Silva, Niki Kilbertus; Stochastic Causal Programming for Bounding Treatment Effect. Proceedings of the Second Conference on Causal Learning and Reasoning, PMLR 213:142-176

What if domain knowledge does not allow you construct a causal diagram?

Data-Driven Covariate Selection for Adjustment

Finding Valid Adjustments under Non-ignorability with Minimal DAG Knowledge

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Abhin Shah, Karthikeyan Shanmugam, and Kartik Ahuja. Finding valid adjustments under non-ignorability with minimal DAG knowledge. In *International Conference on Artificial Intelligence and Statistics (AISTATS - 2022)*, pages 5538–5562. PMLR, 2022.

Differentiable Causal Backdoor Discovery

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The Alan Turing Institute

Varun Kanade
University of Oxford
The Alan Turing Institute

Ricardo Silva
University College London
The Alan Turing Institute

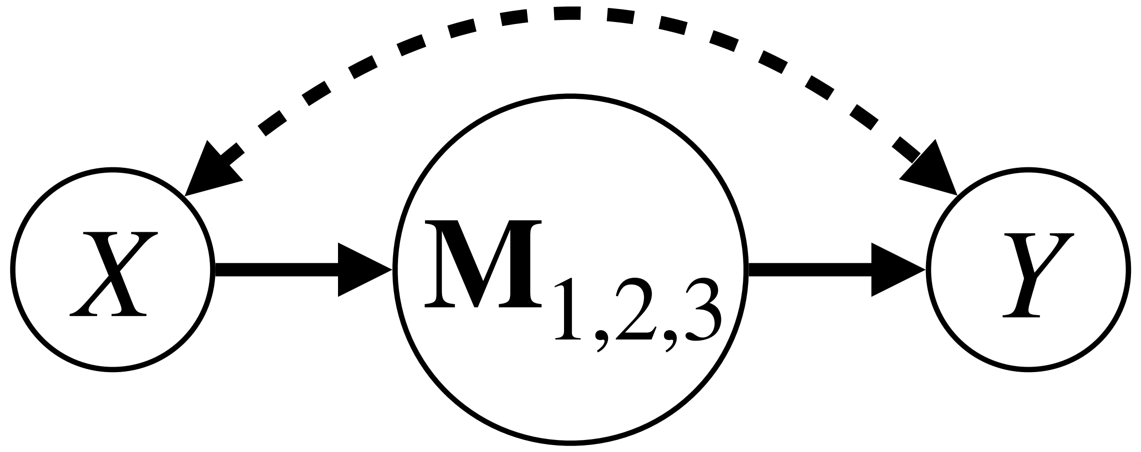
Effect Identification from Cluster DAGs (C-DAGs)

Extension of the ID and Ctf-ID algorithms to C-DAGs.

1. **Query**

   $P(y \mid do(x))$

2. **C-DAG**



3. **Data**

   $P(x, m_1, m_2, m_3, y)$

Inference Engine → **Solution**: yes / no

$$P(y \mid do(x)) = \sum_{\mathbf{m}_{123}} P(\mathbf{m}_{123} \mid x) \sum_{x'} P(y \mid \mathbf{m}_{123}, x')P(x')$$

Inferred (Interventional) Distribution ← Available (Observational) Distribution

Anand, T. V.*, **Ribeiro A. H.*,** Tian, J., & Bareinboim, E. (2023). Causal Effect Identification in Cluster DAGs. In Proceedings of the Thirty-Seventh AAAI Conference on Artificial Intelligence.

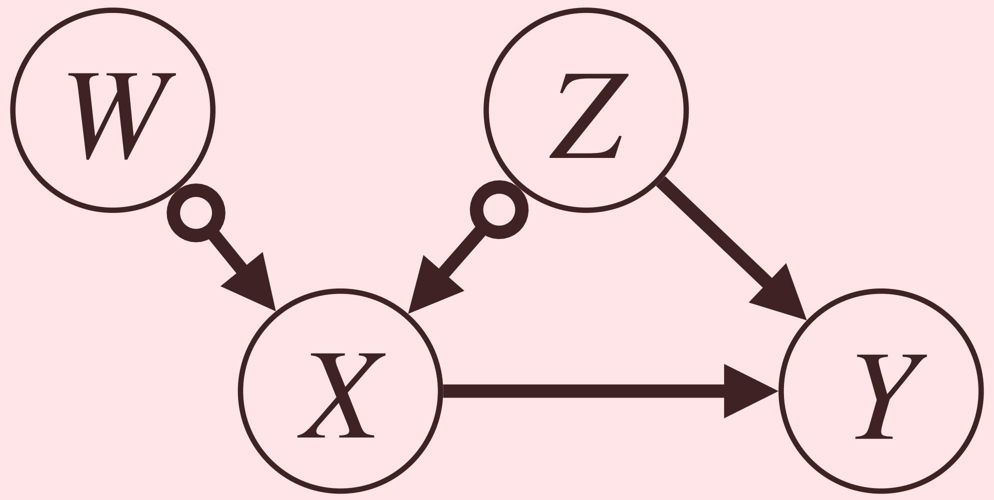
Effect Identification in Markov Equivalence Classes

Can be constructed in a fully data-driven way!

1 **Query**

$P(y \mid do(x))$

2 **PAG**



3 **Data**

$P(x, y, z, w)$ **Observational Distribution**

Inference Engine

Solution

yes / no

$$P(y \mid do(x)) = \sum_{z} P\left(y \mid x, z\right) P\left(z\right)$$

Inferred (Interventional) Distribution ← Available (Observational) Distribution 👍

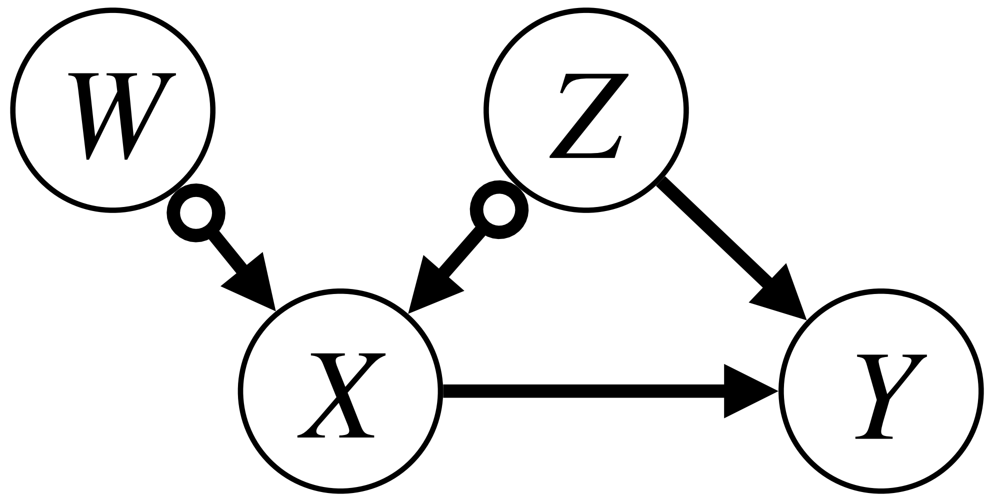
Identification via Adjustment in Markov Equivalence Classes

Identification is possible only when the Generalized Adjustment Criterion applies.

1 **Query**

$P(y \mid do(x))$

2 **PAG**



3 **Data**

$P(x, y, z, w)$ **Observational Distribution**

Adjustment Criterion

Solution

yes / no

$$P(y \mid do(x)) = \sum_z P\left(y \mid x, z\right) P\left(z\right)$$

Inferred (Interventional) Distribution ← Available (Observational) Distribution 👍

Perkovic, E., Textor, J. C., Kalisch, M., & Maathuis, M. H. (2018). Complete graphical characterization and construction of adjustment sets in Markov equivalence classes of ancestral graphs. Journal of Machine Learning Research 18 (2018) 1-62

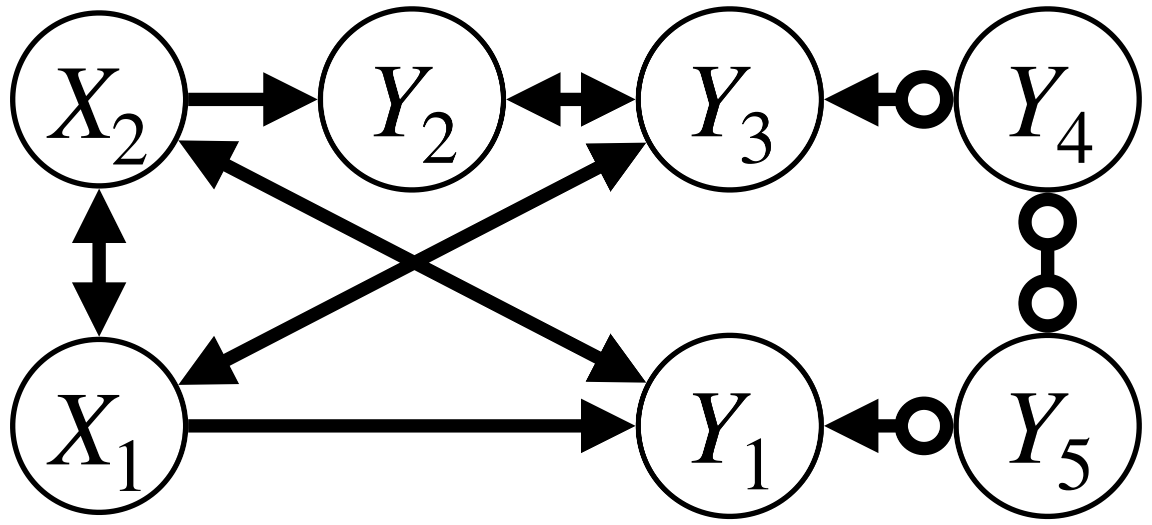
General Identification in Markov Equivalence Classes

Complete algorithms,
available at the *PAGId* R package:
https://github.com/adele/PAGId

1 **Query**

$P(y \mid do(x))$

2 **PAG**



3 **Data**

$P(x, y, z, w)$ **Observational Distribution**

IDP / CIDP

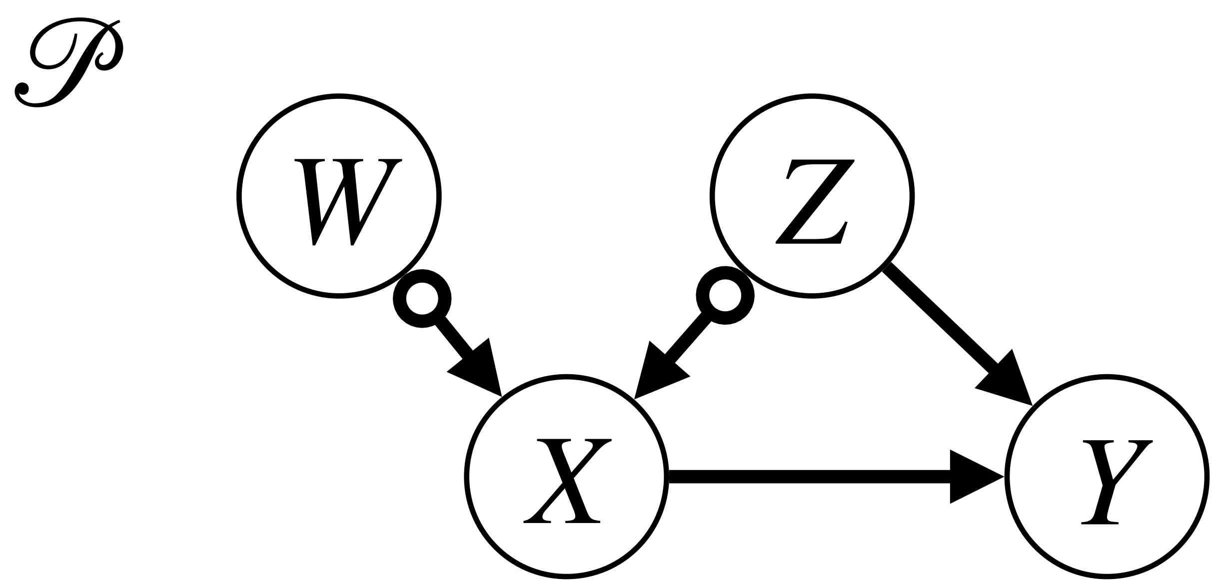
Solution
yes / no

$$P(y \mid do(x)) = \frac{P(y_1, y_4, y_5 \mid x_1) \, . \, P(y_2 y_3, y_4, y_5 \mid x_2)}{P(y_4, y_5)}$$

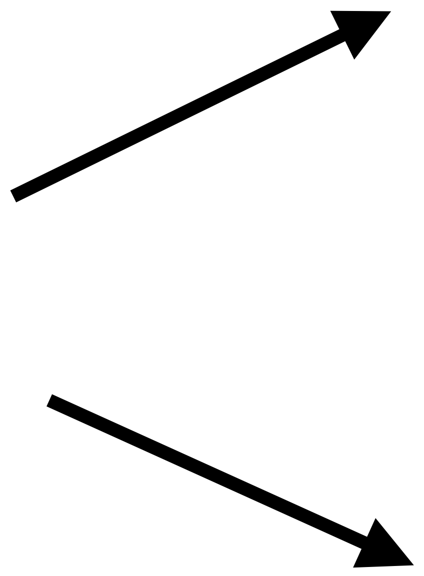
Inferred (Interventional) Distribution ← Available (Observational) Distribution 👍

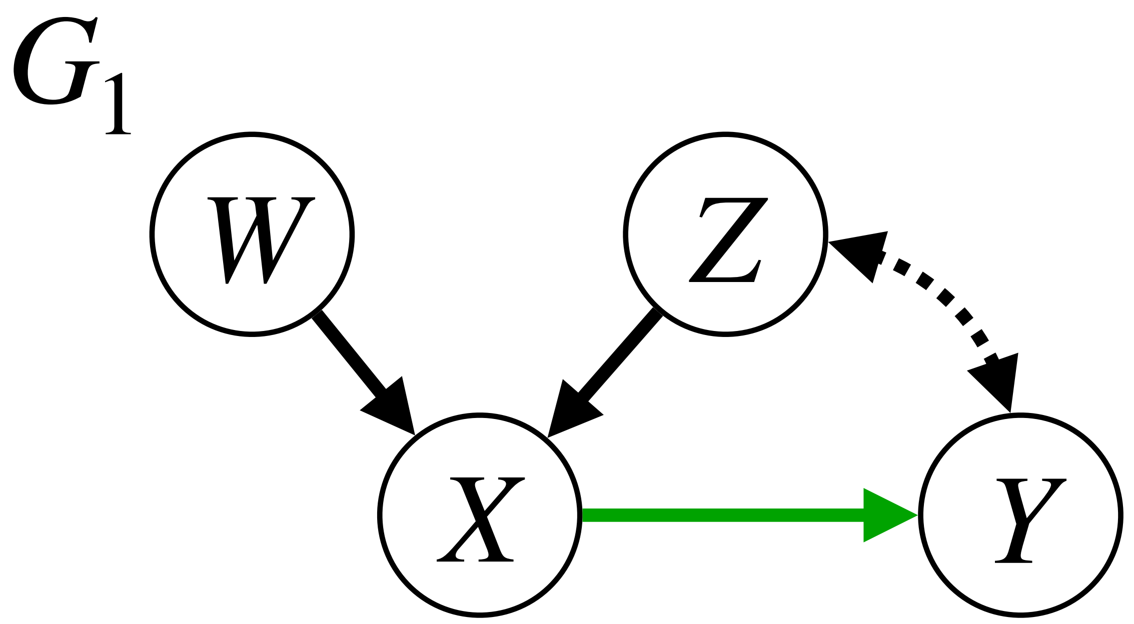
Jaber A., **Ribeiro A. H.,** Zhang, J., Bareinboim, E. (2022) Causal Identification under Markov Equivalence - Calculus, Algorithm, and Completeness. In Proceedings of the 36th Annual Conference on Neural Information Processing Systems (NeurIPS 2022).

Effect Identifiabiliy given a PAG

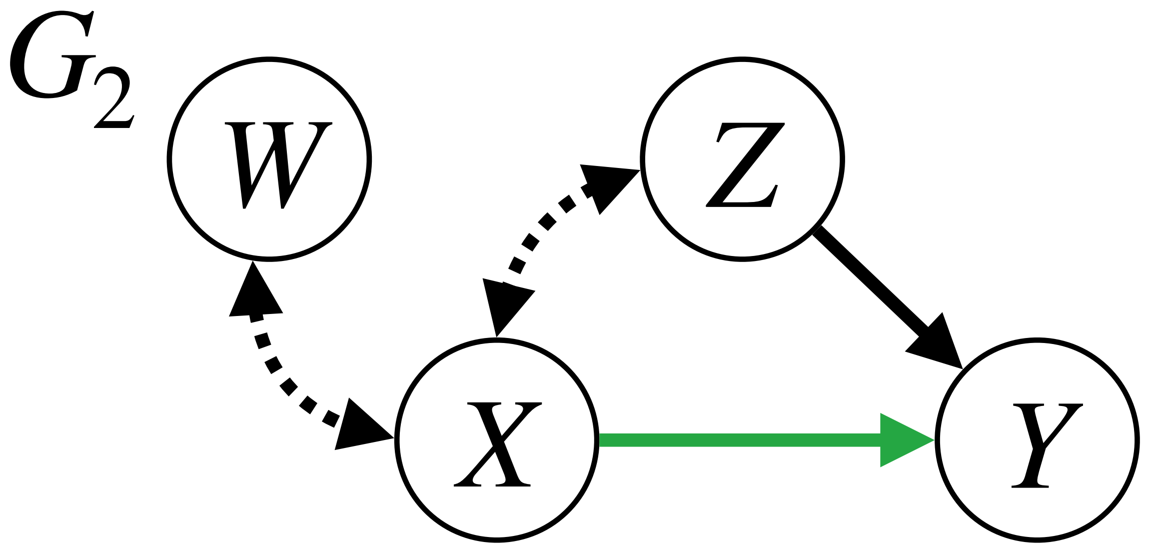


$$P(y \mid do(x)) = \sum_{z} P\left(y \mid x, z\right) P\left(z\right)$$





$$P(y \mid do(x)) = \sum_{z} P(y \mid x, z) P(z)$$

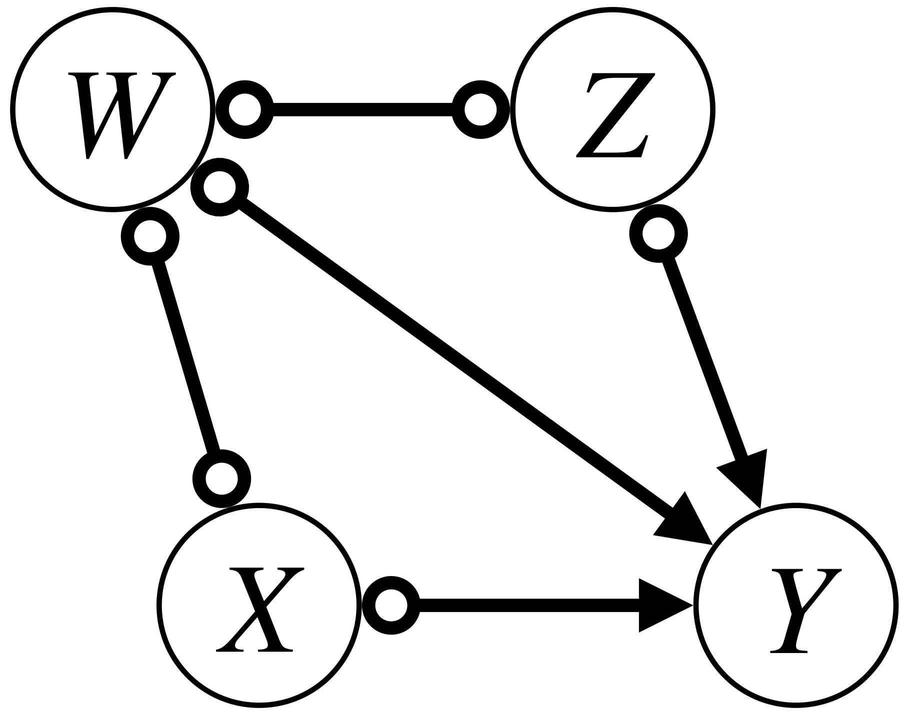


$$P(y \mid do(x)) = \sum_{z} P(y \mid x, z) P(z)$$

An effect identifiable in a PAG $\mathscr{P}$ is identifiable in all causal diagrams G in the Markov Equivalence Class using the same identification formula!

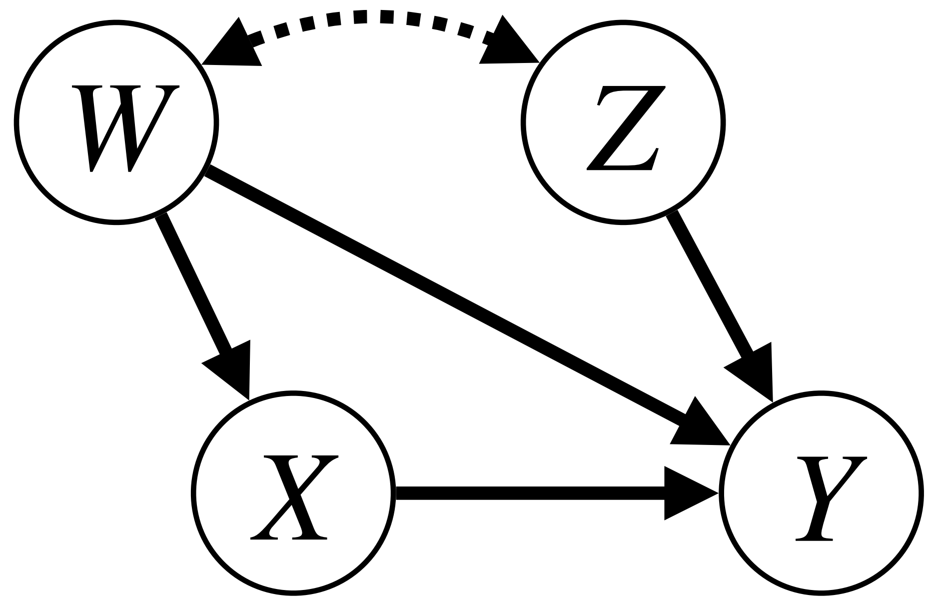
Effect Non-Identifiabiliy given a PAG

$\mathscr{P}$



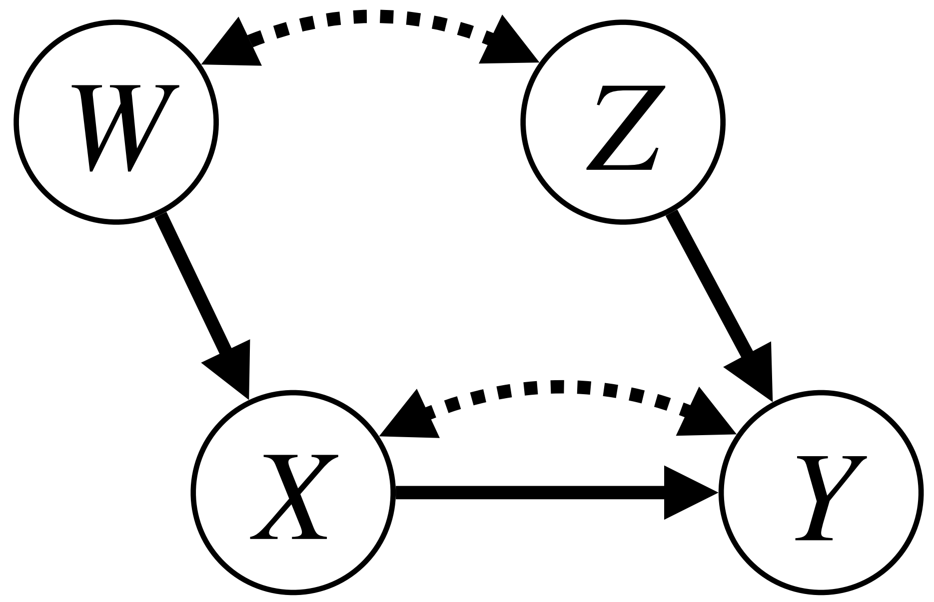
$P(y \mid do(x))$ is
not identifiable

G_1



$$P(y \mid do(x)) = \sum_z P(y \mid x, z) P(z)$$

G_2

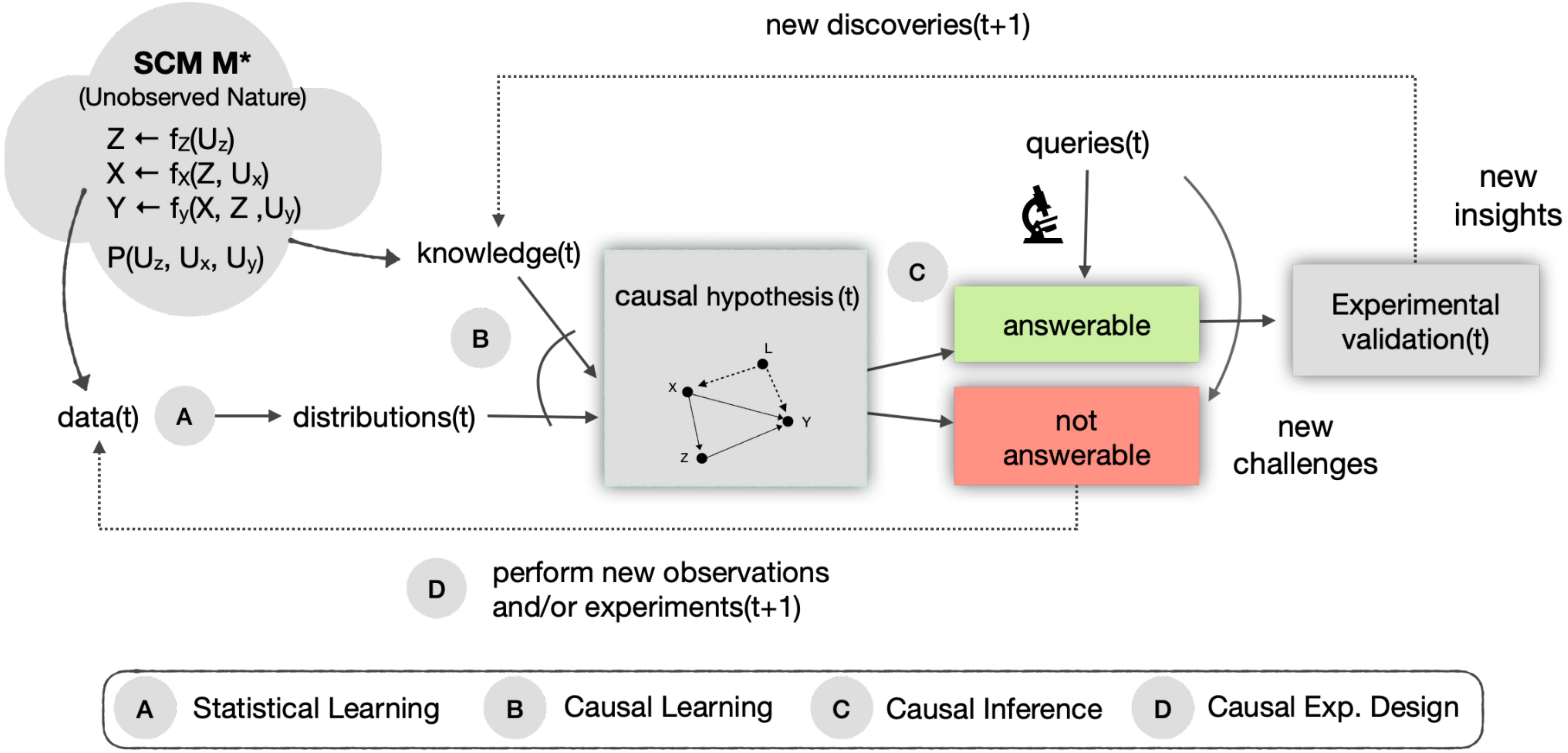


$P(y \mid do(x))$ is
not identifiable

An effect not identifiable in a PAG $\mathscr{P}$ is not identifiable in at least one causal diagrams G in the Markov Equivalence Class

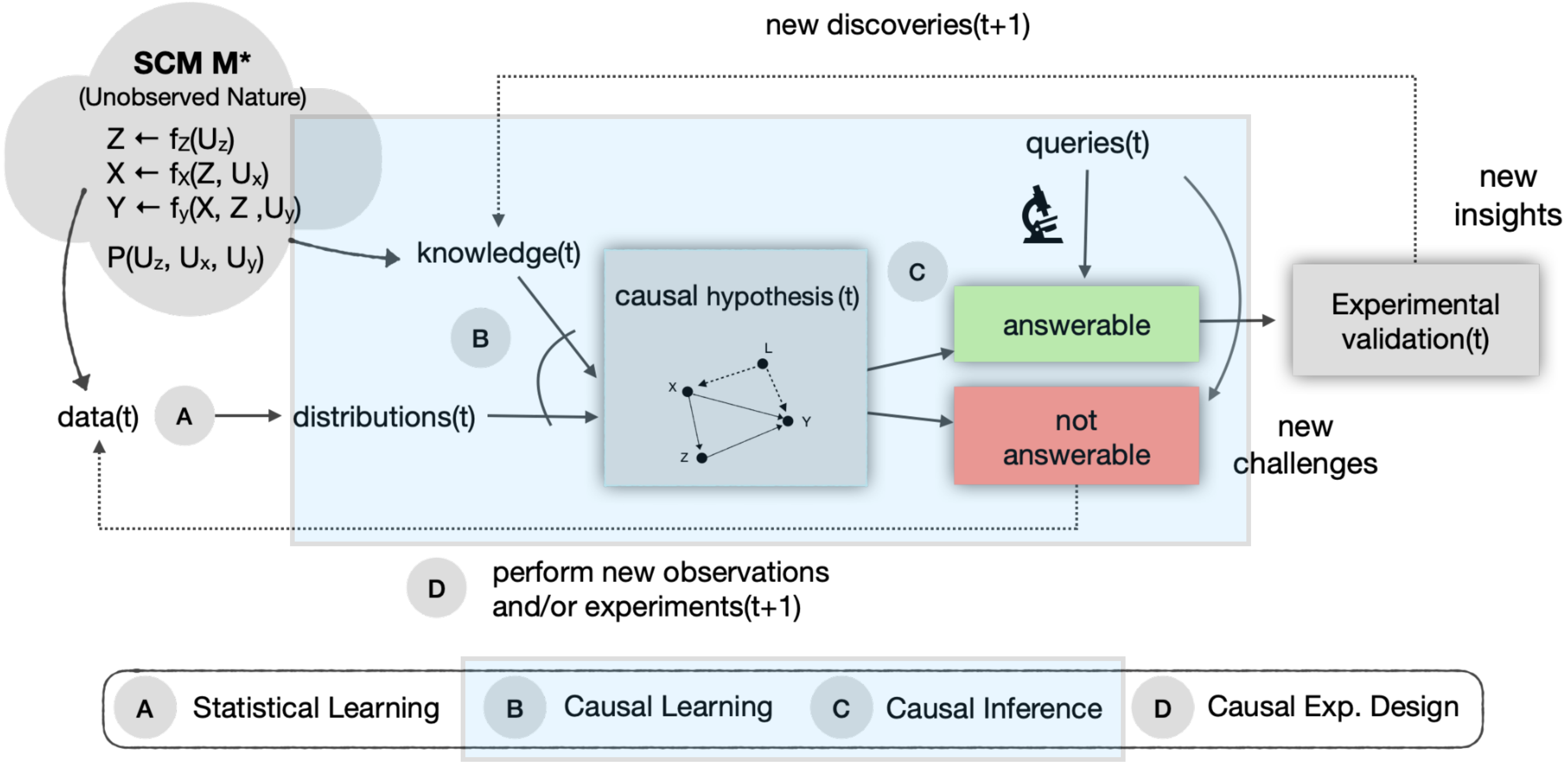
Causal Inference Workflow

Continuous Process of Scientific Discovery and Causal Hypothesis Refinement



Causal Inference Workflow

Continuous Process of Scientific Discovery and Causal Hypothesis Refinement



Current Challenges & Open Problems

- Effect identification in more general equivalence classes.
- Scalability through adaptive, goal-oriented data-driven identification tools.
- Causal effect estimation for general identification formula.
- Causal experimental design — what if a causal effect is not identified?
- Causal effects among abstractions: connection with causal abstraction and causal representation learning.
- Continual Causality - Integrating learning and effect identification

Additional Resources

- Causality Tutorial: https://github.com/adele/Causality-Tutorial/

  → Causal Effect Identification — Google Colab Notebook: (Link)

- Tutorials, talks, and complete lectures on YouTube: (Link)

Feel free to reach out to me if you have any questions or are interested in collaborations.

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Thank you! :)