Elias Fernández Domingos

Outline of the course

- Day 1: Introduction to Game Theory
- Day 2: Evolutionary Game Theory
- **• Day 3-4: Games on Networks, connecting theory to Behavioural Experiments**
- Day 5: Final remarks and Project presentations

Day 3: Games on networks

- 1. Complex networks
- 2. Games on networks

The Evolution of Sectarianism

Sebastian Ille

New College of the Humanities, London

Vítor V. Vasconcelos^{1,2,3}, Francisco C. Santos^{1,3} and Jorge M. Pacheco^{1,4,5} * Human cooperation for reasons other than self-interest has long intrigued social scientists leading to a substantial literature in economics. Its complement – sectarianism – has not received closer attention in economics despite its significant impact. through time¹⁸⁻²¹ (Methods and Supplementary Information for Avoiding the effects of climate change may be framed as a Based on a dynamic model, the paper shows that sectarianism can be understood as further details). Behavioural experiments^{4,5,22}, as well as other theopublic goods dilemma¹, in which the risk of future losses is the outcome of a repeated bargaining process in which sectarian affiliation evolves non-negligible²⁻⁷, while realizing that the public good may be retical models^{23,24}, have implemented thresholds through repeated far in the future^{3,7-9}. The limited success of existing attempts interactions, and other authors have highlighted the role played into a pure coordination signal that attributes economic and political benefits. It by pledges and communication during negotiations^{1,5,25}, bringing to reach global cooperation has been also associated with a demonstrates that such sectarian social contracts co-evolve with the sects' degree about additional layers of complexity to this problem (details and lack of sanctioning institutions and mechanisms to deal with of coerciveness and are self-reinforcing. Sectarian conflict may then not be a result comparison with other models in the Supplementary Information). those who do not contribute to the welfare of the planet or fail to abide by agreements^{1,3,10-13}. Here we investigate the Besides contributing to this public good, Ps also contribute with of diverging religious ideologies but is shown to be caused by external manipulaa punishment tax (π_t) to an institution that, whenever endowed emergence and impact of different types of sanctioning to tions of the signal (e.g. via identity politics), and internal political and economic with enough funding $(n_p \pi_t)$ will effectively punish Ds by an amount deter non-cooperative behaviour in climate agreements. We grievances within a sect that spill over to the inter-sectarian level while adopting a Δ . Hence, establishing an institution stands as a second-order show that a bottom-up approach, in which parties create local ranklin ann 417.20 - third in note patriare data are a postain though ald inatitutiana that numiah fuas sidasa msamataa tha amasanaan sectarian appearance. Theoretical results are supported by empirical findings from the Middle East.

Descriptive modelling framework

nature climate change

TFRS PUBLISHED ONLINE: 14 JULY 2013 | DOI: 10.1038/NCLIMATE1927

A bottom-up institutional approach to cooperative governance of risky commons

LETTER

doi:10.1038/nature25763

Social norm complexity and past reputations in the evolution of cooperation

Fernando P. Santos^{1,2}, Francisco C. Santos^{1,2} & Jorge M. Pacheco^{2,3,4}

Indirect reciprocity is the most elaborate and cognitively demanding \log of all known cooperation mechanisms², and is the most specifically human^{1,3} because it involves reputation and status. By helping someone, individuals may increase their reputation, which may change the predisposition of others to help them in future. The revision of an individual's reputation depends on the social norms that establish what characterizes a good or bad action and thus provide a basis for morality³. Norms based on indirect reciprocity are often sufficiently complex that an individual's ability to follow subjective rules becomes important⁴⁻⁶, even in models that disregard the past reputations of individuals, and reduce reputations to either 'good' or 'bad' and actions to binary decisions^{7,8}. Here we include past reputations in such a model and identify the key pattern in the associated norms that promotes cooperation. Of the norms that comply with this pattern, the one that leads to maximal cooperation (greater than 90 per cent) with minimum complexity does not discriminate on the basis of past reputation; the relative performance of this norm is particularly evident when we consider a 'complexity cost' in the decision process. This combination of high cooperation and low complexity suggests that simple moral principles can elicit cooperation even in

use behavioural strategies (often designated action rules) and strategy spaces that also increase (exponentially with order). For this reason, a combination of a norm and a strategy that promotes cooperation in the space of *n*th-order norms does not necessarily perform equally well in a space of higher-order norms because the availability of more complex behaviours (together with those for lower-order norms) often has non-trivial effects on cooperation¹⁶. Furthermore, the performance of a complex social norm can be constrained by an individual's ability to follow complex subjective rules⁴⁻⁶. This raises two fundamental questions: (1) whether the moral principles that underlie successful strategies and norms in the space of third-order norms remain valid within a larger space, and if so which ones; and (2) how the cognitive skills associated with social norms and strategies impair individuals' performance. Using the donation game and binary reputations we answer these questions by investigating the cooperative capacity of social norms in a space that encompasses norms of up to fourth order and that span a wide range of cognitive complexities^{4,17,18}. Increasing the number of possibilities to consider when assigning a good or a bad reputation to individuals enables us to identify the key pattern of social norms that provides the necessary conditions for promoting cooperation.

Descriptive modelling framework

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**SIMULATION
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ANDTHEORY**

Modeling behavioral experiments on uncertainty and cooperation with population-based reinforcement learning

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Keywords: Public goods game Population dynamics Individual learning Collective risk Uncertainty

ABSTRACT

From climate action to public health measures, human collective endeavors are often shaped by different uncertainties. Here we introduce a novel population-based learning model wherein a group of individuals facing a collective risk dilemma acquire their strategies over time through reinforcement learning, while handling different sources of uncertainty. In such an N-person collective risk dilemma players make step-wise contributions to avoid a catastrophe that would result in a loss of wealth for all players. Success is attained if they collectively reach a certain contribution level over time, or, when the threshold is not reached, they were lucky enough

Descriptive modelling framework

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OPEN Committing to the wrong artificial delegate in a collective-risk dilemma is better than directly committing mistakes

Inês Terrucha $^{1,2\boxtimes}$, Elias Fernández Domingos 2,3,4 , Pieter Simoens 1 & Tom Lenaerts $^{2,3,4,5\boxtimes}$

While autonomous artificial agents are assumed to perfectly execute the strategies they are programmed with, humans who design them may make mistakes. These mistakes may lead to a misalignment between the humans' intended goals and their agents' observed behavior, a problem of value alignment. Such an alignment problem may have narticularly strong consequences when

www.nature.com/scientificreports

SCIENTIFIC REPARTS

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OPEN α -Rank: Multi-Agent Evaluation by **Evolution**

Shayegan Omidshafiel¹, Christos Papadimitriou⁵, Georgios Piliouras \bullet^4 , Karl Tuyls¹, Mark Rowland², Jean-Baptiste Lespiau¹, Wojciech M. Czarnecki², Marc Lanctot³, Julien Perolat² & Remi Munos¹

We introduce α -Rank, a principled evolutionary dynamics methodology, for the evaluation and ranking of agents in large-scale multi-agent interactions, grounded in a novel dynamical game-theoretic solution concept called Markov-Conley chains (MCCs). The approach leverages continuous-time and discrete-time evolutionary dynamical systems applied to empirical games, and scales tractably in the number of agents, in the type of interactions (beyond dyadic), and the type of empirical games (symmetric and asymmetric). Current models are fundamentally limited in one or more of these dimensions, and are not guaranteed to converge to the desired game-theoretic solution concept (typically the Nash equilibrium). α -Rank automatically provides a ranking over the set of agents under evaluation and provides insights into their strengths, weaknesses, and long-term dynamics in terms of basins of attraction and sink components. This is a direct consequence of the correspondence we establish to the dynamical MCC solution concept when the underlying evolutionary model's rankingintensity parameter, α , is chosen to be large, which exactly forms the basis of α -Rank. In contrast to the the property of the company of the company السستمان والمناسب المنوائق التواري

Improve AI self-play in large deep-RL agents

 (b)

(e)

 (b)

1.

Omidshafiei, S., Papadimitriou, C., Piliouras, G., Tuyls, K., Rowland, M., Lespiau, J. B., ... & Munos, R. (2019). α-rank: Multi-agent evaluation by evolution. *Scientific reports*, *9*(1), 9937.

Improve AI self-play in large deep-RL agents

 (a)

(e)

Prescriptive Framework?

Taylor, C., & Nowak, M. A. (2007). Transforming the dilemma. Evolution, 61(10), 2281-2292.

of organization, cooperation is involved. The very origin of life, that are defined by the Prisoner's Dilemma. The payoff matrix of

IREVIEW

Five Rules for the Evolution of Cooperation

Martin A. Nowak

Cooperation is needed for evolution to construct new levels of organization. Genomes, cells, multicellular organisms, social insects, and human society are all based on cooperation. Cooperation means that selfish replicators forgo some of their reproductive potential to help one another. But natural selection implies competition and therefore opposes cooperation unless a specific mechanism is at work. Here I discuss five mechanisms for the evolution of cooperation: kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection. For each mechanism, a simple rule is derived that specifies whether natural selection can lead to cooperation.

volution is based on a fierce competition well-mixed populations needs help for establishbetween individuals and should therefore
resources in the cooperation.
Treward only selfish behavior. Every gene, every cell, and every organism should be designed to promote its own evolutionary success When J. B. S. Haldane remarked. "I will jump ical organization. Genes cooperate in genomes. as Hamilton's rule (1). This ingenious idea is that $(12-14)$. Chromosomes cooperate in eukaryotic cells. natural selection can favor cooperation if the cooperation is the decisive organizing principle of human society. No other life form on Earth is engaged in the same complex games of cooper ation and defection. The question of how natural selection can lead to cooperative behavior has fascinated evolutionary biologists for several decades.

A cooperator is someone who pays a cost, became widely known as "kin selection" or c , for another individual to receive a benefit, b. A defector has no cost and does not deal (Fig. 1). Therefore, selection acts to increase the relative abundance of defectors. After some Direct Reciprocity

 $r > c/b$ Relatedness is defined as the probability of sharing a gene. The probability that two brothers

Kin Selection

Nowak, M. A. (2006). Five rules for the evolution of cooperation. science, 314(5805), 1560-1563.

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Review

Feature Review

TICS-1212; No. of Pages 13

Human cooperation

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Why should you help a competitor? Why should you contribute to the public good if free riders reap the benefits of your generosity? Cooperation in a competitive world is a conundrum. Natural selection opposes the evolution of cooperation unless specific mechanisms are at work. Five such mechanisms have been proposed: direct reciprocity, indirect reciprocity, spatial selection, multilevel selection, and kin selection. Here we discuss empirical evidence from laboratory experiments and field studies of human interactions for each mechanism. We also consider cooperation in one-shot, anonymous interactions for which no mechanisms are apparent. We argue that this behavior reflects the overgeneralization of cooperative strategies learned in the context of direct and indirect reciprocity: we show that automatic, intuitive responses favor cooperative strategies that reciprocate.

defection [1]. These interaction structures specify how the individuals of a population interact to receive payoffs, and how they compete for reproduction. Previous work has identified five such mechanisms for the evolution of cooperation (Figure 1): direct reciprocity, indirect reciprocity, spatial selection, multilevel selection, and kin selection. It is important to distinguish between interaction patterns that are mechanisms for the evolution of cooperation and behaviors that require an evolutionary explanation (such as strong reciprocity, upstream reciprocity, and parochial altruism; Box 2).

In this article, we build a bridge between theoretical work that has proposed these mechanisms and experimental work exploring how and when people actually cooperate. First we present evidence from experiments that implement each mechanism in the laboratory. Next we discuss why cooperation arises in some experimental settings in which no machanisms are annarent Finally we consider the

The challenge of cooperation

Rand, D. G., & Nowak, M. A. (2013). Human cooperation. Trends in cognitive sciences, 17(8), 413-425.

viduals or even between members of different species. Such considerations led Trivers (10) to propose another mechanism for the evolution of cooperation, direct reciprocity. Assume that there are reneated encounters between the same two individuals. In every round, each player has a choice between cooperation and defection. If I cooperate now, you may cooperate later. Hence, it might pay off to cooperate. This game theoretic framework is known as the repeated Prisoner's Dilemma

observe cooperation between unrelated indi-

But what is a good strategy for playing this game? In two computer tournaments, Axelrod (11) discovered that the "winning strategy" was the simplest of all, tit-for-tat. This strategy always starts with a cooperation, then it does whatever the other player has done in the previous round: a cooperation for a cooperation, a defection for a defection. This simple concept captured the fascination of all enthusiasts of the repeated Prisoner's Dilemma. at the expense of its competitors. Yet we ob- into the river to save two brothers or eight Many empirical and theoretical studies were serve cooperation on many levels of biolog-cousins," he anticipated what became later known inspired by Axelrod's groundbreaking work

But soon an Achilles heel of the world Cells cooperate in multicellular organisms. There donor and the recipient of an altruistic act are champion was revealed: If there are erroneous are many examples of cooperation among ani-
genetic relatives. More precisely, Hamilton's rule moves caused by "trembling hands" or "fuzzy mals. Humans are the champions of cooperation: states that the coefficient of relatedness, r, must minds," then the performance of tit-for-tat de-From hunter-oatherer societies to nation-states. exceed the cost-to-benefit ratio of the altruistic act: clines (15, 16). Tit-for-tat cannot correct mistakes, because an accidental defection leads to a long sequence of retaliation. At first, tit-for-tat was replaced by generous-tit-for-tat (17) , a strategy that cooperates whenever you cooperate, but sometimes cooperates although you have share the same gene by descent is 1/2; the same defected [with probability $1 - (c/b)$]. Natural probability for cousins is 1/8. Hamilton's theory selection can promote forgiveness. Subsequently, tit-for-tat was replaced b

 (1)

"inclusive fitness" $(2-7)$. When evaluating the win-stay, lose-shift, which is the even simpler fitness of the behavior induced by a certain gene, idea of repeating your previous move whenout benefits. Cost and benefit are measured in it is important to include the behavior's effect on ever you are doing well, but changing otherterms of fitness. Reproduction can be genetic kin who might carry the same gene. Therefore, wise (18) . By various measures of success, or cultural. In any mixed population, defectors the "extended phenotype" of cooperative behav- win-stay, lose-shift is more robust than either and have a higher average fitness than cooperators ior is the consequence of "selfish genes" $(8, 9)$. itt-for-tat or generous-tit-for-tat $(15, 18)$. Titfor-tat is an efficient catalyst of cooperation in a \overrightarrow{S} society where nearly everybody is a defector, $\sum_{k=1}^{\infty}$

Four of the five rules

kin selection direct reciprocity

indirect reciprocity group selection

Rule number five

Part 1: Complex networks

Some good references

Statistical physics of human cooperation

illian J. Jordan, 3 David G. Rand, $3, 4, 5$ Zhen Wang, 6 Stefano Boccaletti, $7, 8$ a iral Sciences and Mathematics, University of Maribor, Koroška cesta 160, SI-2000 2 CAMTP – Center for Applied Mathematics and Theoretical Physics, University of Maribor, Mladinska 3, SI-2000 Maribor, Slovenia ³Department of Psychology, Yale University, New Haven, Connecticut 06511, US ⁴Department of Economics, Yale University, New Haven, Connecticut 06511, US ³ School of Management, Yale University, New Haven, Connecticut 06511, USA itical Imagery Analysis and Learning, Northwestern Polytechnical University, Xi'a titute of Complex Systems, Via Madonna del Piano, 10, 50019 Sesto Fiorentino, I ⁸The Italian Embassy in Israel, 25 Hamered st., 68125 Tel Aviv, Israel ⁹Institute of Technical Physics and Materials Science, Centre for Energy Researc

Hungarian Academy of Sciences, P.O. Box 49, H-1525 Budapest, Hungary

cooperation among unrelated individuals is unique to humans, who often sacrifice p non good and work together to achieve what they are unable to execute alone. T ur species is indeed due, to a large degree, to our unparalleled other-regarding a ve understanding of human cooperation remains a formidable challenge. Recent re cates that it is important to focus on the collective behavior that emerges as the res ng individuals, groups, and even societies. Non-equilibrium statistical physics, in part ds and the theory of collective behavior of interacting particles near phase transit very valuable for understanding counterintuitive evolutionary outcomes. By study eration as classical spin models, a physicist can draw on familiar settings from stat like pairwise interactions among particles that typically govern solid-state physics g humans often involve group interactions, and they also involve a larger number of most simplified description of reality. The complexity of solutions therefore ofter physical systems. Here we review experimental and theoretical research that ad of human cooperation, focusing on spatial pattern formation, on the spatiotempor utions, and on self-organization that may either promote or hinder socially favorable

Statistical Mechanics of Complex Networks

- Réka Albert^{1,2} and Albert-László Barabási²
-
-

Complex networks describe a wide range of systems in nature and society, much quoted examples including the cell, a network of chemicals linked by chemical reactions, or the Internet. a network of routers and computers connected by physical links. While traditionally these systems were modeled as random graphs, it is increasingly recognized that the topology and evolution of real networks is governed by robust organizing principles. Here we review the recent advances in the field of complex networks, focusing on the statistical mechanics of network topology and dynamics. After reviewing the empirical data that motivated the recent interest in networks, we discuss the main models and analytical tools, covering random graphs, small-world and scale-free networks, as well as the interplay between topology and the network's robustness against failures and attacks.

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II. THE TOPOLOGY OF REAL NETWORKS: **EMPIRICAL RESULTS**

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- III. RANDOM GRAPH THEORY A. The Erdős-Rényi model
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Physics Reports 424 (2006) 175-308

Complex networks: Structure and dynamics

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> Accepted 27 October 2005 Available online 10 January 2006 editor: I. Procaccia

iological and chemical systems, neural networks, social interacting species, the Internet and the V v examples of systems composed by a large number of highly interconnected dynamical units. The lobal properties of such systems is to model them as graphs whose nodes represent the dynamical r the interactions between them. On the one hand, scientists have to cope with structural issues, such If a complex wiring architecture, revealing the unifying principles that are at the basis of real network. nic the growth of a network and reproduce its structural properties. On the other hand, many releval g complex networks' dynamics, such as learning how a large ensemble of dynamical systems that i ig topology can behave collectively. We review the major concepts and results recently achieved if dynamics of complex networks, and summarize the relevant applications of these ideas in many difnonlinear science to biology, from statistical mechanics to medicine and engineering. ier B.V. All rights reserved.

Some good references

The Science of Complex Contagions

What are social networks?

Ego Network

Social Ego Network

taken from Centola 2018

What are social networks?

taken from Centola 2018

What are social networks?

<https://www.youtube.com/watch?v=4fHufyIWmX0>

Complexity
Theory 8

https://www.youtube.com/watch?v=-ckaLBsCoxo&t=1s

- A graph G can be defined as $G = (V, E)$ where V is the set of vertices or **nodes** of the graph, and **E** the set of edges connecting every two nodes in the graph.
- We can also represent a finite graph using an **adjacency matrix A**. This $n \times n$ square matrix indicate whether pairs of vertices in the graph are connected by an edge, i.e., every $A_{ij} = 1$ where there is and edge between nodes v_i and v_j , otherwise $A_{ij} = 0$.
- Graphs can be **directed** or **undirected**. In undirected graphs, the adjacency matrix is symmetric.

•

• The edges of a graph can also be weighted, i.e., some edges are more important than

• The **distance matrix** is a weighted adjacency matrix, and the **distance** between two

- others.
- nodes $d(v_i, v_j)$ in the network can be defined as the minimum sum of the sum of the **along the shortest path connecting them.**

weights on the shortest path between two nodes. Or simply, for binary networks (nonweighted) **the distance between two nodes is defined as the number of edges**

•

- The degree (k) of a node is the total number of edged incident on that node in a binary undirected network.
-
- In weighted networks, the strength measure is also considered.

•

• In directed networks, we differentiate between in-degree (k_{in}) and out-degree (k_{out}) .

• A **clique** is a subset of nodes of an undirected graph (or network) such that every two distinct nodes in the clique are adjacent. That is, a clique in a graph G is a complete

subgraph of G.

•

Important concepts: Small worlds

Watts, Duncan J., and Steven H. Strogatz. "Collective dynamics of 'small-world'networks." *nature* 393.6684 (1998): 440-442.

There are many important measures and indicators of a network topology. We will focus on the following three concepts:

- **Small worlds**
- **Clustering**
- **• Degree distribution**

Important concepts: Small worlds

Important concepts: Clustering

- **Clustering**: cliques tend to form in social networks, representing circles of close friends. This effect can be quantified using the **clustering coefficient** (Watts and Strogatz 1998).
- Let's assume we have a node i in the network, with k_i edges which connect it to k_i other nodes. If the first neighbours of the original node were part of a clique, there would be $k_{i}(k_{i}-1)/2$ edges between them.
- The ratio between the number E_i of edges that actually exist between these k_i nodes and the total number of nodes in a clique $k_i(k_i-1)/2$ gives the value of the clustering coefficient of node : *i*

$$
C_i = \frac{2E_i}{k_i(k_i - 1)}
$$

The clustering coefficient of the whole network is an average of all individual C_i 's.

Important concepts: Clustering

Important concepts: Clustering

Fig. 2.3. Communities can be defined as groups of nodes such that there is a higher density of edges within groups than between them. In the case shown in figure there are three communities, denoted by the dashed circles. Reprinted figure with permission from Ref. [51]. © 2004 by the American Physical Society.

Boccaletti et al., 'Complex Networks'.

Important concepts: Degree distribution

Degree distribution: The spread in the number of edges a node has, or node degree, is characterized by the distribution function $P(k)$.

 $P(k)$ gives the probability that a randomly selected node has exactly k edges.

Characterization and Traversal of Large **Real-World Networks**

A. Garcia-Robledo, ... G. Morales-Luna, in Big Data, 2016

5.3 Characterization and Measurement

A complex network $G = V$, E is a non-empty set V of nodes or vertices and a set E of links or edges, such that there is a mapping between the elements of E and the set of pairs $\{i, j\}$, $i, j \in V$. Let $n = V$ be the number of vertices and $m = E$ be the number of edges of G. The degree k_i of a vertex $i \in V$ is the number of neighbors of *i*. Let n_k be the number of vertices of degree k in G, such that $\sum_{k} n_k = n$. Let $Pk = n_k / n$ be the degree distribution of G.

Complex networks, random graphs, and graphs arising in scientific computing (e.g., meshes and lattices) are all sparse. However, unlike these kinds of graphs, complex networks present the combined

https://www.sciencedirect .com/science/article/pii/B 9780128053942000052

Structure and social ties

https://www.youtube.com/watch? v=sl8TK2mETrk&source_ve_path=MjM4NTE

Regular Networks

$$
\begin{aligned} \langle k \rangle &= 4\\ P(k) &= \delta(4) \end{aligned}
$$

Complex Networks

- Random graphs
- Small world
- Scale free

Complex Networks: Erdös-Rényi (ER) Random Network

- The Erdös-Rényi model defines a random graph as N labeled nodes connected by n edges which are chosen randomly from the $N(N-1)/2$ possible edges.
- This defines $\frac{1+\sqrt{1+\frac{1}{2}}}{2}$ possible graphs with N nodes and n edges. A random network can be generating by choosing one of these possible graphs with equal probability *Cn N*(*N*−1 2

Complex Networks: Erdös-Rényi (ER) Random Network

- Alternatively, we can construct a random graph using what is known as the **binomial model**.
- \bullet Start with N nodes, and then connect every pair of nodes with probability p .
- In this case, the total number of edges is a random variable with expectation $E(n) = pN(N-1)/2$. Thus, the probability of obtaining a graph with N nodes and n edges is $P(N, n) = e^{n}(1-p)^{(N(N-1)/2 - n)}$. $\Big($ *N*(*N* − 1)/2 − *n*)
- The degree distribution $P(k)$ of a random graph is a Poisson distribution with a peak at P ($\langle k \rangle$).

Complex Networks: Erdös-Rényi (ER) Random Network

.

Albert and Barabasi, 'Statistical Mechanics of Complex Networks'.

 $p=0.15$

Complex Networks: Erdös-Rényi (ER) Random Network

 $P(k) \simeq e^{-pN \frac{(pN)}{k!}}$ *k* $\frac{d^{k}k!}{k!} = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$ *k*!

Albert and Barabasi, 'Statistical Mechanics of Complex Networks'.

FIG. 7. The degree distribution that results from the numerical simulation of a random graph. We generated a single random graph with $N = 10,000$ nodes and connection probability $p = 0.0015$, and calculated the number of nodes with degree k, X_k . The plot compares X_k/N with the expectation value of the Poisson distribution (13), $E(X_k)/N = P(k_i = k)$, and we can see that the deviation is small.

Complex Networks: Small-World Networks

- Also known as the Wattz and Strogatz model, proposed in (Wattz & Strogatz 1998).
- Algorithm:
	- 1. Start with order: start with a ring lattice with N nodes in which every node is connected to its first K neighbours ($K/2$ on either side). In order to have a sparse, but connected network at all times, consider $N \gg K \gg ln(N) \gg 1$.
	- 2. **Randomise**: Randomly rewire each edge of the lattice with probability p, such that self-connections and duplicate edges are excluded. This process introduces *pNk*/2 long-range edges which connect to nodes that otherwise would be part of different neighbourhoods. p gives control over the transition between order (regular lattice) and full randomness.

Complex Networks: Small-World Networks

Regular

 $P_r \cong 0$

Saadat, Yalda, et al. 2018

Small-world

Random

 $P_r \cong 1$

Complex Networks: Small-World Networks

Albert and Barabasi, 'Statistical **Mechanics of Complex Networks'.**

 $(2000).$

FIG. 19. Degree distribution of the WS model for $K = 3$ and various p. We can see that only $k \geq K/2$ values are present, and the mean degree is $\langle k \rangle = K$. The symbols are obtained from numerical simulations of the WS model with $N = 1000$, and the lines correspond to Eq. (76) . As a comparison, the degree distribution of a random graph with the same parameters is plotted with filled symbols. After Barrat and Weigt

A Random network

Brigandt, Ingo, Sara Green, and Maureen A. O'Malley 2017.

B Scale-free network

- Graph degree distribution follows a power law $P(k) \sim Ak^{-\lambda}$.
- It was found that many of the real-world networks display a degree distribution that is shaped as a power law with exponents varying in the range $2 < \lambda < 3.$

Barabási and Albert (1999) argued that the scale-free nature of real networks is rooted in two

generic mechanisms common in many real networks:

- **Growth**: most real networks grow by continuously attaching new nodes to a small nucleolus.
- **• Preferential attachment:** the likelihood of connecting to a node depends on the node's degree.

Algorithm:

- 1. Growth: Growth: starting with a small number (m_0) of nodes, at every time-step we add a new node with m ($\leq m_0$) edges that link the new node to m different nodes already present in the system.
- 2. **Preferential attachment**: when choosing the nodes to which the new node connects, we assume that the probability P that a new node will be connected to node depends on the degree k_i such that $P(k_i) = \frac{1}{\sum_{i} k_i}$. *ki* $\sum_j k_j$

- After t time-steps this algorithm results in a network with $N = t + m_0$ nodes and mt edges.
- Numerical simulations indicate that this network evolves into a scale-invariant state with the probability that a node has k edges following a power-law with an exponent $\lambda_{SF} = 3.$

Albert and Barabasi, 'Statistical **Mechanics of Complex Networks'.**

 $10⁻²$ P(K) 10^4 (a) $10⁻⁶$ 10

FIG. 21. (a) Degree distribution of the scale-free model, with $N = m_0 + t = 300,000$ and $m_0 = m = 1$ (circles), $m_0 = m = 3$ (squares), $m_0 = m = 5$ (diamonds) and $m_0 = m = 7$ (triangles). The slope of the dashed line is $\gamma = 2.9$. The inset shows the rescaled distribution (see text) $P(k)/2m^2$ for the same values of m, the slope of the dashed line being $\gamma = 3$. (b) $P(k)$ for $m_0 = m = 5$ and system sizes $N = 100,000$ (circles), $N = 150,000$ (squares) and $N = 200,000$ (diamonds). The inset shows the time-evolution for the degree of two vertices, added to the system at $t_1 = 5$ and $t_2 = 95$. Here $m_0 = m = 5$, and the dashed line has slope 0.5 , as predicted by Eq. (80) . After Barabási, Albert, Jeong (1999).

Complex Networks: Real-world examples

Albert and Barabasi, 'Statistical Mechanics of Complex Networks'.

1999)

Complex Networks: how to infer the network topology from data?

In finite-size networks, fat-tailed degree distributions have natural cut-offs [83]. When analyzing real networks, it may happen that the data have a rather strong intrinsic noise due to the finiteness of the sampling. Therefore, when the size of the system is small and the degree distribution $P(k)$ is heavy-tailed, it is sometimes advisable to measure the *cumulative* degree distribution $P_{\text{cum}}(k)$, defined as $P_{\text{cum}}(k) = \sum_{k'=k}^{\infty} P(k')$. Indeed, when summing up the original distribution $P(k)$, the statistical fluctuations generally present in the tails of the distribution are smoothed. Consequently, if $P(k) \sim k^{-\gamma}$, the exponent γ can be obtained from $P_{\text{cum}}(k)$ as one plus the slope of $P_{\text{cum}}(k)$ in a log-log plot, i.e., $\gamma = 1 + \gamma_{\text{cum}}$. Another possibility is that of performing an exponential binning of data [8].

Boccaletti et al., 'Complex Networks: structure and dynamics'.

numuwa wa pwwag wwimuwuu wannumum

Complex Networks: how to infer the network topology from data?

Fig. 2.4. Cumulative degree distributions of the Internet AS graph representation for three different years. The power-law behavior is clear, as well as the fact that, regardless of the very dynamic nature of the Internet, the exponent γ is constant with time. Reprinted figure with permission from Ref. [25]. © 2001 by the American Physical Society.

Boccaletti et al., 'Complex Networks: structure and dynamics'.

Complex Networks: how to infer the network topology from data?

Use Maximum Likelihood (MLE) to fit distribution like the Power Law, not the Least-Squares (LSE)!

If the estimation errors belong to a normal distribution, then MLE are LSE, but this does not have to be true for other distributions.

https://www.youtube.com/watch?v=UdADuHJUX6Q

Network diffusion and contagion

[https://www.researchgate.net/profile/Badziili-Nthubu/publication/333902908/figure/fig2/](https://www.researchgate.net/profile/Badziili-Nthubu/publication/333902908/figure/fig2/AS:772952572301312@1561297662054/Visualisation-of-weak-ties-vs-strong-ties-IE-A-link-to-IE-C-represents-a-weak-tie-which.jpg) [AS:772952572301312@1561297662054/Visualisation-of-weak-ties-vs-strong-ties-IE-A-link-to-IE-C-represents-a-weak-tie](https://www.researchgate.net/profile/Badziili-Nthubu/publication/333902908/figure/fig2/AS:772952572301312@1561297662054/Visualisation-of-weak-ties-vs-strong-ties-IE-A-link-to-IE-C-represents-a-weak-tie-which.jpg)[which.jpg](https://www.researchgate.net/profile/Badziili-Nthubu/publication/333902908/figure/fig2/AS:772952572301312@1561297662054/Visualisation-of-weak-ties-vs-strong-ties-IE-A-link-to-IE-C-represents-a-weak-tie-which.jpg)

Network diffusion and contagion

Part 2: Games on networks

Levels of abstraction

Levels of abstraction

Structured populations: Spatial games

- Spatial structure among plants or animals in an ecosystem
- The graph can also describe the architecture of cells in a multicellular organism, including the cellular differentiation hierarchy
- Relationships in a social network

• Dynamics on graph describe cultural evolution and the spread of new inventions and ideas

•

Some good references

Evolutionary dynamics of social dilemmas in structured heterogeneous populations

F. C. Santos*, J. M. Pacheco^t, and Tom Lenaerts*¹⁵

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Edited by Brian Skyrms, University of California, Irvine, CA, and approved December 15, 2005 (received for review September 21, 2005)

Real populations have been shown to be heterogeneous, in which some individuals have many more contacts than others. This fact contrasts with the traditional homogeneous setting used in studies of evolutionary game dynamics. We incorporate heterogeneity in the population by studying games on graphs, in which the variability in connectivity ranges from single-scale graphs, for which heterogeneity is small and associated degree distributions exhibit a Gaussian tale, to scale-free graphs, for which heterogeneity is large with degree distributions exhibiting a power-law behavior.

Original Paper

Learning to coordinate in complex networks

Sven Van Segbroeck^{1,2}, Steven de Jong^{1,3}, Ann Nowé¹,
Francisco C Santos⁴ and Tom Lenaerts^{1,2}

Adaptive Behavior 18(5) 416-427 C The Author(s) 2010 Reprints and permissions: sagepub.co.uk/journalsPermissions.nav DOI: 10.1177/1059712310384282 adb.sagepub.com \$SAGE

Adaptive Behavior

Abstract

Designing an adaptive multi-agent system often requires the specification of interaction patterns between the different agents. To date, it remains unclear to what extent such interaction patterns influence the dynamics of the learning mechanisms inherent to each agent in the system. Here, we address this fundamental problem, both analytically and via computer simulations, examining networks of agents that engage in stag-hunt games with their neighbors and thereby learn to coordinate their actions. We show that the specific network topology does not affect the game strategy the agents learn on average. Yet, network features such as heterogeneity and clustering

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PLOS COMPUTATIONAL BIOLOGY

Cooperation Prevails When Individuals Adjust **Their Social Ties**

Francisco C. Santos¹, Jorge M. Pacheco^{2,3}, Tom Lenaerts^{4,5*}

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Conventional evolutionary game theory predicts that natural selection favours the selfish and strong even though cooperative interactions thrive at all levels of organization in living systems. Recent investigations demonstrated that a limiting factor for the evolution of cooperative interactions is the way in which they are organized, cooperators becoming evolutionarily competitive whenever individuals are constrained to interact with few others along the edges of networks with low average connectivity. Despite this insight, the conundrum of cooperation remains since recent empirical data shows that real networks exhibit typically high average connectivity and associated single-to-broadceala bataraganaitu. Hara a computational modal ic conctructed in which individuale are able to celf-organize both

PRL 108, 158104 (2012)

PHYSICAL REVIEW LETTERS

week ending 13 APRIL 2012

Emergence of Fairness in Repeated Group Interactions

S. Van Segbroeck,¹ J. M. Pacheco,^{2,3} T. Lenaerts,^{1,4} and F. C. Santos^{5,3} ¹MLG, Université Libre de Bruxelles, Brussels, Belgium ²Departamento de Matemática e Aplicações, Universidade do Minho, Braga, Portugal ³ATP-group, CMAF, Instituto para a Investigação Interdisciplinar, Lisboa, Portugal ⁴AI-lab, Vrije Universiteit Brussel, Brussels, Belgium ⁵DEI, & INESC-ID, Instituto Superior Técnico, TU Lisbon, Lisboa, Portugal (Received 26 August 2011; published 10 April 2012)

Often groups need to meet repeatedly before a decision is reached. Hence, most individual decisions will be contingent on decisions taken previously by others. In particular, the decision to cooperate or not will depend on one's own assessment of what constitutes a fair group outcome. Making use of a repeated N-person prisoner's dilemma, we show that reciprocation towards groups opens a window of opportunity for cooperation to thrive, leading populations to engage in dynamics involving both coordination and coexistence, and characterized by cycles of cooperation and defection. Furthermore, we show that this process leads to the emergence of fairness, whose level will depend on the dilemma at stake.

DOI: 10.1103/PhysRevLett.108.158104

PACS numbers: 87.23.Kg, 89.75.Fb

Suppressors of selection

Lieberman, Erez, Christoph Hauert, and Martin A. Nowak. "Evolutionary dynamics on graphs." *Nature* 433.7023 (2005): 312-316.

Amplifiers of selection

Lieberman, Erez, Christoph Hauert, and Martin A. Nowak. "Evolutionary dynamics on graphs." *Nature* 433.7023 (2005): 312-316.

superstar funnel

Proc. Natl. Acad. Sci. USA Vol. 79, pp. 1331-1335, February 1982
Population Biology

Assortment of encounters and evolution of cooperativeness

(altruism/evolutionary stable strategies/assortative meetings)

ILAN ESHEL[†] AND L. L. CAVALLI-SFORZA

Departments of Mathematics and Genetics, Stanford University, Stanford, California 94305

Contributed by L. L. Cavalli-Sforza, October 13, 1981

The method of evolutionary stable strategies **ABSTRACT** (ESS), in its current form, is confronted with a difficulty when it tries to explain how some social behaviors initiate their evolution. We show that this difficulty may be removed by changing the assumption made tacitly in game theory (and in ESS) of randomness

In the case of nonrandom encounters due to active individuals may actively seek or avoid encounters wit individuals of their phenotype or strategy. These choic be the result of learning by the individual, or they may netically or culturally inherited traits that have spread

Nowak, M. A., May, R. M., & Sigmund, K. (1995). The arithmetics of mutual help. Scientific American, 272(6), 76-81.

Homogeneous interactions in space

Red is a D who was a D before Blue is a C who was a C before Green is a C who was a D before Yellow is a D who was a C before

Nowak, M. A., & May, R. M. (1992). Evolutionary games and spatial chaos. nature, 359(6398), 826-829.

Spatial games

The fraction of C **stabilises** over time in the grid

Games on networks

What is the network structure?

How are strategies updated?

Can individual change their social ties?

Neighbourhood

contribute. The total contribution is multiplied by an enhancement

Santos, Santos, and Pacheco, 'Social Diversity Promotes the Emergence of Cooperation in Public Goods Games'.

sence of spatial and network reciprocity.........

Figure 1 | Population structure and local neighbourhoods. a, Regular graphs studied so far, which mimic spatially extended systems. b, Scale-free graphs⁹ in which small-world effects coexist with a large heterogeneity in neighbourhood size. c, The focal individual (largest sphere) belongs to different groups (neighbourhoods) of different sizes in a heterogeneous graph. Given his/her connectivity $k = 4$, we identify five neighbourhoods, each centred on one of the members of the focal individual's group, such that individual fitness derives from the payoff accumulated in all five neighbourhoods $(\alpha, \beta, \gamma, \delta$ and $\varepsilon)$.

Common rules for behavioural update/adaptation

There are many…, but the most common ones are:

1. "Birth-death"

This process can also be **synchronous** or **asynchronous**

Common rules for behavioural update/adaptation

There are many…, but the most common ones are:

2. "Death-Birth"

This process can also be **synchronous** or **asynchronous**

Common rules for behavioural update/adaptation

There are many…, but the most common ones are:

3. Imitation or social learning

This process can also be **synchronous** or **asynchronous**

The role of heterogeneity

S \blacktriangle ╱ h.

Evolutionary dynamics of social dilemmas in structured heterogeneous populations

F. C. Santos*, J. M. Pacheco^t, and Tom Lenaerts*^{‡§}

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Edited by Brian Skyrms, University of California, Irvine, CA, and approved December 15, 2005 (received for review September 21, 2005)

Real populations have been shown to be heterogeneous, in which some individuals have many more contacts than others. This fact contrasts with the traditional homogeneous setting used in studies of evolutionary game dynamics. We incorporate heterogeneity in the population by studying games on graphs, in which the variability in connectivity ranges from single-scale graphs, for which heterogeneity is small and associated degree distributions exhibit a Gaussian tale, to scale-free graphs, for which heterogeneity is large with degree distributions exhibiting a power-law behavior.

D

D

 $k_{10} = 1$

Modelling evolution on networks

Vertex x plays k_{χ} times and **accumulates** payoff *Fx*

Simulating stochastic evolutionary dynamics

Choose a neighbour *y* with payoff *Fy*

Replace strategy s_x in node x by strategy $s_{\rm y}$ of node ${\rm y}$ with probability

$$
p = \max\left[0, \frac{F_y - F_x}{k_y - S}\right]
$$

Well-mixed, the baseline

Regular networks

T

Random (Erdös-Rényi) networks

Scale-free (Barabasi) networks

T

Randomized Scale-free (Barabasi) networks

T

Punish

Sucker **<** Punish

Transforming the PD in an SH or SD

Nowak, M. A. (2006). Five rules for the evolution of cooperation. *science*, *314*(5805), 1560-1563. 78

Sucker

OPEN & ACCESS Freely available online

PLos one

From Local to Global Dilemmas in Social Networks

Flávio L. Pinheiro¹, Jorge M. Pacheco^{1,2}, Francisco C. Santos^{1,3}*

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Abstract

Social networks affect in such a fundamental way the dynamics of the population they support that the global, populationwide behavior that one observes often bears no relation to the individual processes it stems from. Up to now, linking the global networked dynamics to such individual mechanisms has remained elusive. Here we study the evolution of cooperation in networked populations and let individuals interact via a 2-person Prisoner's Dilemma – a character heterogeneous networks promote their coordination. To this end, we define a dynamic variable that allows us to track the self-organization of cooperators when co-evolving with defectors in networked populations. Using the same variable, we show how the global dynamics $-$ and effective dilemma $-$ co-evolves with the motifs of cooperators in the population, the overall emergence of cooperation depending sensitively on this co-evolution.

Citation: Pinheiro FL, Pacheco JM, Santos FC (2012) From Local to Global Dilemmas in Social Networks. PLoS ONE 7(2): e32114. doi:10.1371/journal.pone.0032114 Editor: James A. R. Marshall, University of Sheffield, United Kingdom

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sts: The authors have declared that no competing interests exist

Introduces a new tool: the **averaged gradient of selection**

Assortment leads to the transformation of the game

homogeneous random network Scale-Free

Pinheiro, F. L., Pacheco, J. M. & Santos, F. C. From Local to Global Dilemmas in Social Networks. *PLoS ONE* **7**, e32114 (2012). 79

(Barrabassi-Albert)

Increasing heterogeneity favours cooperation

Pinheiro F L, Pacheco J M and Santos F C 2012 From Local to Global Dilemmas in Social Networks *PLoS One* **7** e32114

- 1. Heterogeneity in structure also leads to heterogeneous payoffs among individuals (even if they adopt the same strategy), since some individuals interact more often than others
- 2. **Local** information may be different from **global**

https://github.com/Socrats/EGTTools

Domingos, E. F., Santos, F. C., & Lenaerts, T. (2023). EGTtools: Evolutionary game dynamics in Python. Iscience, 26(4): 106419 [https://doi.org/10.1016/](https://doi.org/10.1016/j.isci.2023.106419) [j.isci.2023.106419](https://doi.org/10.1016/j.isci.2023.106419)

EGTtools demo

iScience

Article

EGTtools: Evolutionary game dynamics in Python

What about n-player games?

Vol 454 10 July 2008 doi:10.1038/nature06940

Social diversity promotes the emergence of cooperation in public goods games

Francisco C. Santos¹, Marta D. Santos² & Jorge M. Pacheco²

Humans often cooperate in public goods games $1-3$ and situations ranging from family issues to global warming^{4,5}. However, evolutionary game theory predicts^{4,6} that the temptation to forgo the public good mostly wins over collective cooperative action, and this is often also seen in economic experiments⁷. Here we show how social diversity provides an escape from this apparent paradox. Up to now, individuals have been treated as equivalent in all respects^{4,8}, in sharp contrast with real-life situations, where diversity is ubiquitous. We introduce social diversity by means of heterogeneous graphs and show that cooperation is promoted by the diversity associated with the number and size of the public goods game in which each individual participates and

nature

$IFRS$

factor r and the result is equally distributed between all N members of the group. Hence, Ds get the same benefit of the Cs at no cost. Collective action to shelter, protect and nourish, which abounds in the animal world, provides examples of PGGs, because the cooperation of group members is required. Ultimately, the success (and survival)⁵ of the human species relies on the capacity of humans for large-scale cooperation. In the absence of enforcement mechan- $\text{isms}^{7,13,14}$, conventional evolutionary game theory predicts that the temptation to defect leads individuals to forgo the public good⁴ in the N-person prisoner's dilemma. Whenever interactions are not repeated, and reward and punishment^{4,8,13} can be ruled out, several mechanisms were explored that promote cooperation. Individuals

What about n-player games?

Public good game

Heterogeneous networks also promote cooperation in public good games

Fixed cost per game

Fixed cost per individual

And non-linear games?

Risk of collective failure provides an escape from the tragedy of the commons

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Edited by Simon A. Levin, Princeton University, Princeton, NJ, and approved May 11, 2011 (received for review October 18, 2010)

contain at least $M Cs$ (or equivalently, a collective effort of Mcb), From group hunting to global warming, how to deal with all members will lose their remaining endowments with a probcollective action may be formulated in terms of a public goods game of cooperation. In most cases, contributions depend on the ability r (the risk); otherwise, everyone will keep whatever they risk of future losses. Here, we introduce an evolutionary dynamics have. Imposing such a threshold mimics situations common to approach to a broad class of cooperation problems in which most of the public endeavors described above, and it also extends attempting to minimize future losses turns the risk of failure into to nonhuman dilemmas $(21-23)$, where a minimum combined a central issue in individual decisions. We find that decisions effort is needed to achieve a collective goal. This is also the case within small groups under high risk and stringent requirements to in international environmental agreements (extensive reviews in success significantly raise the chances of coordinating actions and refs. 5, 6, 11, and 12), which demand a minimum number of escaping the tragedy of the commons. We also offer insights on ratifications to come into practice (3, 24, 25). the scale at which public goods problems of cooperation are best Rational players facing this one-shot dilemma will ont for

Heterogeneity also fosters cooperation in the Collective Risk Dilemma

Fixed threshold

Variable threshold

^aInstituto de Biocomputación y Física de Sistemas Complejos (BIFI), Universidad de Zaragoza, 50018 Zaragoza, Spain; ^bDepartamento de Física Teórica, Universidad de Zaragoza, 50009 Zaragoza, Spain; and ^cGrupo Interdisciplinar de Sistemas Complejos (GISC), Departamento de Matemáticas, Universidad Carlos III de Madrid, 28911 Leganés, Madrid, Spain

Edited by Simon A. Levin, Princeton University, Princeton, NJ, and approved June 8, 2012 (received for

evolutionary game dynamics | network reciprocity | conditional cooperation

with some except \bigcap α account the behavior extracted from extracted from extracted from extracted from extracted from extracted from α . predicts that neith would influence t_0
lemma (i.e., the cooperative behavior in the Prisoner's D.6 lemma (i.e., the \circ as if every player \bullet

Here, we close \blacksquare dictions (19) and istence and effects of ~ 0.2 experiments on late viduals who interaction
Specifically, we have.
 $\mathbf{0}_{\mathsf{D}}^{\mathsf{L}}$ subjects were pla network, and for multiplayer PD game $\sum_{n=1}^{\infty}$ negation for the integration of the integration of the integration on $\sum_{n=1}^{\infty}$ one action [either multaneously carried 25×25 lattice with subjects) and a heterogeneous 0.6 distribution (604 subsetsion of ϵ tween $k = 2$ and $k = 16$. Fig. 1.1 and $k = 16$. representation of more details on the experimental setup as $\begin{bmatrix} 0.2 \end{bmatrix}$ the actions of the SI Materials and 1

et al. (15) on a smaller regular lattice (Fig. 1), and there is a smaller regular lattice (Fig. 1), and therefore, cooperation should reach a high level according to the available level according to the available to th

the players were always connected to the same number of \mathbf{r} bors , but the neighbors changed from round the round to round the round to round the round to reduce the round to round the round term in the reduce of antives avnouimantally **Problematic to confirm experimentally**

It is not fully understood why we cooperate with strangers on a daily basis. In an increasingly global world, where interaction networks and relationships between individuals are becoming more complex, different hypotheses have been put forward to explain the foundations of human cooperation on a large scale and to account for the true motivations that are behind this phenomenon. In this context, population structure has been suggested to foster cooperation in social dilemmas, but theoretical studies of this mechanism have yielded contradictory results so far; additionally, the issue lacks a proper experimental test in large systems. We have performed the largest experiments to date with humans playing a spatial Prisoner's Dilemma on a lattice and a scale-free network (1,229 subjects). We observed that the level of cooperation reached in both networks is the same, comparable with the level of cooperation of smaller networks or unstructured populations. We have also found that subjects respond to the cooperation that they observe in a reciprocal manner, being more likely to cooperate if, in the previous round, many of their neighbors and themselves did so, which implies that humans do not consider neighbors' payoffs when making their decisions in this dilemma but only their actions. Our results, which are in agreement with recent theoretical predictions based on this behavioral rule, suggest that population structure has little relevance as a cooperation promoter or inhibitor among humans.

The strong cooperative attitude of humans defies the paradigm of Homo economicus and poses an evolutionary conundrum

(1, 2). This conundrum is because many of our interactions can

The histograms (C and D) show the fraction of subjects ranked actions that they perform along the experiment in th

for any player facing a defector (weak PD) (9). We note that this choice of payoffs is like the experiment in the work by Gruen by Grupi choice \mathcal{B}/\mathcal{E} 87

Heterogeneous networks do not promote cooperation when humans play a Prisoner's Dilemma

Carlos Gracia-Lázaro^a, Alfredo Ferrer^a, Gonzalo Ruiz^a, Alfonso Tarancón^{a,b}, José A. Cuesta^{a,c}, Angel Sánchez^{a,c,1}, and Yamir Morenoa,b,1

Reinforcement learning = individual learning by experience (see part 2)

RESEARCH ARTICLE

Reinforcement Learning Explains Conditional Cooperation and Its Moody Cousin

Takahiro Ezaki^{1,2,3,4}, Yutaka Horita^{3,4}, Masanori Takezawa^{5,6}, Naoki Masuda⁷*

1 Research Center for Advanced Science and Technology, The University of Tokyo, Meguro-ku, Tokyo, Japan, 2 Japan Society for the Promotion of Science, Kojimachi, Chiyoda-ku, Tokyo, Japan, 3 National Institute of Informatics, Hitotsubashi, Chiyoda-ku, Tokyo, Japan, 4 JST, ERATO, Kawarabayashi Large Graph Project, c/o Global Research Center for Big Data Mathematics, NII, Chiyoda-ku, Tokyo, Japan, 5 Department of Behavioral Science, Hokkaido University, Kita-ku, Sapporo, Japan, 6 Center for Experimental Research in Social Sciences, Hokkaido University, Kita-ku, Sapporo, Japan, 7 Department of Engineering Mathematics, University of Bristol, Clifton, Bristol, United Kingdom

* naoki.masuda@bristol.ac.uk

Abstract

Direct reciprocity, or repeated interaction, is a main mechanism to sustain cooperation under social dilemmas involving two individuals. For larger groups and networks, which are Citation: Ezaki T, Horita Y, Takezawa M, Masuda Nulle and engineering our society, experiments employ-
Cooperation and Its Moody Cousin. PLoS Comput
Cooperation and Its Moody Cousin. PLoS Comput
Conditional cooperation beh behaviors largely remain unclear. Here we provide a proximate account for this behavior by showing that individuals adopting a type of reinforcement learning, called aspiration learnina indhanananaalaalaha kahaya aa aandiilanal aaanavatav. Du dafinitian indhiiduda ava

Social Experiments in the Mesoso **Spatial Prisoner's Dilemma**

Jelena Grujić¹, Constanza Fosco^{1¤}, Lourdes Araujo², José A.

1 Grupo Interdisciplinar de Sistemas Complejos (GISC), Departamento de Matemáticas, Unive Procesamiento de Lenguaje Natural (NLP and IR), Departamento de Lenguajes y Sistemas, UNED, UCM, Cantoblanco, Madrid, Spain, 4 Instituto de Biocomputación y Física de Sistemas Complejos

Abstract

Background: The evolutionary origin of cooperation among unrelated several disciplines. Prominent among the several mechanisms proposed existence of a population structure that determines the interactions analytically and by simulation the effects of such a structure, particularly the results of these models largely depend on details such as the type Therefore, experimental work suitably designed to address this questior

Methods and Findings: We have designed an experiment to test the Prisoner's Dilemma on a network whose size is comparable to that of declines to an asymptotic state with low but nonzero cooperation. R population is heterogeneous, consisting of a high percentage of defector that shares features of the conditional cooperators of public goods game: coexistence of these different strategies that is in good agreement with

Conclusions: In our large experimental setup, cooperation was not prome level (around 20%) typical of public goods experiments. Our findings also

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Biol 12(7): e1005034. doi:10.1371/journal. pcbi.1005034

Editor: Natalia L. Komarova, University of California,

Individual learning (alone) has no effect in networks

What is the effect of changing topologies?

OPEN O ACCESS Freely available online

Cooperation Prevails When Individuals Adjust **Their Social Ties**

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Conventional evolutionary game theory predicts that natural selection favours the selfish and strong even though cooperative interactions thrive at all levels of organization in living systems. Recent investigations demonstrated that a limiting factor for the evolution of cooperative interactions is the way in which they are organized, cooperators becoming evolutionarily competitive whenever individuals are constrained to interact with few others along the edges of networks with low average connectivity. Despite this insight, the conundrum of cooperation remains since recent empirical data shows that real networks exhibit typically high average connectivity and associated single-to-broadceala hataraanaaitu. Hara a camnutatianal madal ic canctructad in which individuale ara ahla ta calf arganiya hath

Both numerical and analytical approaches have been proposed

PLOS COMPUTATIONAL BIOLOGY

How to rewire ?

- The first can rewire with probability *p* to a neighbour of the other D.

C wants to change the link with D. This rewire with probability p to a neighbour of D

- The second can rewire with probability $(1-p)$

$$
p = (1 + e^{-\beta (F_a - F_b)})^{-1}
$$

D wants to change the link with D.

Time-scale differences

High average degree
$$
\hat{k} = 30
$$

 $Z = 10^4, 100$ runs,

Time scale and degree are linked

Time-scale differences

Confirmation …

LETTER

promotes human co

Katrin Fehl, Daniel J. van der Post and Dirk Semmann* Junior Research Group Evolution of Cooperation and Prosocial Behaviour, Courant Research Centre

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Abstract

favoured by natural selection. To links after each social interaction. Our results highlight the importa cooperation.

Keywords

social behaviour.

Ecology Letters (2011) 14: 546–551

INTRODUCTION

Cooperative behaviour is widespread throughout the animal

Co-evolution of behaviour and social network structure Dynamic social networks promote cooperation in experiments with humans

David G. Rand^{a,b,1}, Samuel Arbesman^{c,d,1}, and Nicholas A. Christakis^{c,d,e,f,2}

The ubiquity of cooperation in nervous and the collectionary bynamics, bepartment of Psychology, institute for Quantitative social science, and bepartment of sociology, narvard university, Cambridge, MA 02138; and ^dDepar ^aProgram for Evolutionary Dynamics, ^bDepartment of Psychology, ^cInstitute for Quantitative Social Science, and ^eDepartment of Sociology, Harvard

theoretical work shows that if a stated by Douglas S. Massey, Princeton University, Princeton, NJ, and approved October 18, 2011 (received for review May 23, 2011)

repeated games between participar **in the latter organized. Human interactions are not random but rather are** structure ra Through biased link breaking (i.e. to defect on the symmetry manging in response to the senation of our social conditions of the social partners. This dynamic structure permits an important form procity is conditions. this link-breaking behaviour leads of conditional action that has been explored theoretically but has pend on y these clusters. This assortment is **referred the empirical attention. Febre can respond to the co**rrection of the direct reciprocity and beyond the perception of preaking network links. Here, we present experimental evidence tat" strate Assortment, co-evolution, cooper are shuffled randomly every round or are fixed across all rounds. The cooperation of the coop based on social networks of non-human primates show that the show th collective action | economic games | evolutionary game theory | exercise in the structure and a mutu Human populations are both highly cooperative and highly structured in social networks. Importantly, ties in these networks often are dynamic, changing in response to the behavior of one's received little empirical attention: People can respond to the coof the power of using strategic link formation and dissolution, and the network modification it entails, to stabilize cooperation in sizable groups. Our experiments explore large-scale cooperation, where subjects' cooperative actions are equally beneficial to all those with whom they interact. Consistent with previous research, we find that cooperation decays over time when social networks We also find that, when networks are dynamic but are updated only infrequently, cooperation again fails. However, when subjects can update their network connections frequently, we see a qualitatively different outcome: Cooperation is maintained at a high level through network rewiring. Subjects preferentially break links with defectors and form new links with cooperators, creating an incentive to cooperate and leading to substantial changes in network structure. Our experiments confirm the predictions of a set of evolutionary game theoretic models and demonstrate the important role that dynamic social networks can play in supporting large-scale human cooperation. conditional action, one that occurs via changes in network structure rather than via changes in cooperation behavior. Behavioral reciprocity is a central mechanism for the evolution of cooperation (1, 20, 21). In evolutionary game theory, reciprocity is defined as occurring when my actions toward you depend on your actions in the past. Reciprocity traditionally has been conceptualized in two-player game theory as the emergence of concordant behaviors within dyads. For example, the "tit-fortat" strategy engages in reciprocity by cooperating only if the opponent cooperated in the previous round. Reciprocity creates future consequences for one's choices and has been shown experimentally to promote cooperation in repeated two-player interactions (22–25). However, reciprocity is problematic in group interactions involving more than two players: If the only way to sanction defectors is to defect, this action also harms the other cooperators in one's group (26). Strategic tie formation and dissolution in dynamic networks offer a solution to this problem by providing players with an additional method of responding to the past actions of others. Players can reciprocate not only by changing their cooperation behaviors but also by creating or dissolving ties. Thus, cooperators need not switch to defection to punish defectors in their group; instead they can establish and maintain links with cooperators but sever connections with defectors, engaging in what we call "link reciprocity." (Note that this reciprocity is different from the use of the term in social network analysis, where reciprocity refers to the existence of tie concordance in directed graphs—that is, if ego nominates alter, alter also nominates ego, and a mutually reciprocated tie evicts.

& Kasper 2009).

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In recent years, a number of evolutionary game theory models have demonstrated the ability of link reciprocity to promote the

ECOLOGY LETTERS

But, do people really use social learning on networks?

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Research

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Do people imitate when making decisions? Evidence from a spatial Prisoner's Dilemma experiment

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Humans do imitate more successful individuals on a local scale

Grujić, J., & Lenaerts, T. (2020). Do people imitate when making decisions? Evidence from a spatial prisoner's dilemma experiment. *Royal Society open science*, *7*(7), 200618.

TWO: Treatment without payoff difference information

TWI: Treatment with payoff difference information

Is this still relevant?

Hindawi Complexity Volume 2021, Article ID 6851477, 11 pages https://doi.org/10.1155/2021/6851477

WILEY

Research Article

Eliciting Fairness in N-Player Network Games through **Degree-Based Role Assignment**

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Evolutionary Game Theoretic Insights on the SIRS Model of the COVID-19 Pandemic Madeo D. * Mocenni C. *

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Abstract: The effectiveness of control measures against the diffusion of the COVID-19 pandemic is grounded on the assumption that people are prepared and disposed to cooperate. From a strategic decision point of view, cooperation is the unreachable strategy of the prisoner's dilemma game, where the temptation to exploit the others and the fear to be betrayed by them drives the people behavior, which eventually results fully defective. In this work, we integrate the SIRS epidemic model with the replicator equation of evolutionary games in order to study the interplay between the infection spreading and the propensity of people to become cooperative under the pressure of the epidemic. We find that the developed model possesses several steady states, including fully or partially cooperative ones and that the presence of such states allows to take the disease under control. Moreover, assuming a seasonal variation of the infection rate, the system presents rich dynamics, including chaotic behavior and epidemic extinction.

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Is this still relevant?

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Frontiers

Cooperation dynamics under pandemic risks ar economic interdependence

Network Diversity Promotes Safety Adoption in Swift Artificial Intelligence Development

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Abstract

Regulating the development of advanced technology such as Artificial Intelligence (AI) has become a principal topic, given the potential threat they pose to humanity's long term future. First deploying such technology promises innumerable benefits, which might lead to the disregard of safety precautions or societal consequences in favour of speedy devel-

which innovation dynamics are pictured through the lens of Evolutionary Game Theory (EGT) and where all race participants are equally well-connected in the system. The baseline results have showed the importance of accounting for different time-scales of development, and also exposed the dilemmas that arise when what is individually preferred by developers differs from what is globally beneficial. How-

Is this still relevant?

Link recommendation algorithms and dynamics of polarization in online social networks

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The level of antagonism between political groups has risen in the That is not an easy task. As pointed out by Woolley and Howard, "to understand contemporary political communication past years. Supporters of a given party increasingly dislike memwe must now investigate the politics of algorithms and automabers of the opposing group and avoid intergroup interactions, leading to homophilic social networks. While new connections tion" (16). While traditional media outlets are curated by huoffline are driven largely by human decisions, new connections on mans, online social media resorts to computer algorithms to online social platforms are intermediated by link recommendation personalize contents through automatic filtering. To understand algorithms, e.g., "People you may know" or "Whom to follow" information dynamics in online social networks, one needs to suggestions. The long-term impacts of link recommendation in potake into account the interrelated subtleties of human decision larization are unclear, particularly as exposure to opposing viewmaking [e.g., only share specific contents (17), actively engage points has a dual effect: Connections with out-group members can with other users, follow or befriend particular individuals, inlead to opinion convergence and prevent group polarization or teract offline] and the outcomes of automated decisions (e.g., further separate opinions. Here, we provide a complex adaptivenews sorting and recommendation systems) (18, 19). In this systems perspective on the effects of link recommendation algoregard, much attention has been placed on the role of news filrithms. While several models justify polarization through rewiring ters and sorting $(1, 18, 19)$. Shmargad and Klar (20) provide based on opinion similarity, here we explain it through rewiring

FEATURE SPECIAL I

JTER SCIENCES

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Particular Constitutions ?